Do Frictions Matter in the Market for Chief Executives?

Lorán Chollete*

Irina Merkurieva†

This version: December 30, 2020

Abstract

The chief executive officers (CEOs) of the public companies receive generous compensation packages that grow substantially faster than the general wages, contributing to the increasing income disparities. To understand the causes of this growth, we develop and estimate a dynamic equilibrium model of executive employment and pay that accommodates search frictions, headhunting, counter-offers and the cost of CEO replacement. Search frictions decrease welfare by 40% relative to the benchmark frictionless equilibrium. Wage growth is mainly driven by the ability of incumbents to extract additional match surplus. Headhunting explains approximately 25% of the steady state growth of executive compensation. Increase in the market capitalization of the public companies that is thought to explain most of the executive compensation growth in the static assignment models can only account for about one third of the wage growth observed over the last decade.

Keywords: Executive Compensation; Headhunting; Mismatch; Replacement Cost; Search Frictions; Search-Matching; Sorting; Wage Growth

JEL classification: D31, D83, J3, J62, J63, J64, M12

*Welch College of Business and Technology, Sacred Heart University, Fairfield, CT, USA 06825. email cholletel@sacredheart.edu. Chollete acknowledges funding for this project from Finansmarkedsfondet Grant # 230876 and the Research Council of Norway.

†School of Economics and Finance, University of St. Andrews, Castlecliffe, The Scores, St. Andrews, UK, KY169AR. E-mail: i.merkurieva@st-andrews.ac.uk.

1 Introduction

The largest U.S. publicly traded firms (S&P 500) are the key players in a variety of industries ranging from information technology to utilities. They affect all essential areas of the economy and account for 13% of the total employment. The efficiency of their corporate governance, including decisions on the compensation of their chief executive officers (CEOs), has important implications on many levels.

Generous compensation packages received by the CEOs of the major companies increasingly draw public attention in the context of widening income inequality. In 2017, the S&P 500 executives earned on average $11.7 million, tenfold the average income of the top 0.5 percent households. Over a period of eight years since the end of the Great Recession, the real pay of the top executives has grown by 18 percent, three times faster than the real average salary in the U.S. private sector. This sustained fast growth of executive compensation would have contributed to the doubling of income shares held by the wealthiest households that occurred since the 1980s.[1]

In this paper we estimate to what extent the growth of executive compensation can be attributed to market frictions. Our results are based on the dynamic equilibrium model of executive employment and wages that stems from the search and matching framework of Mortensen and Pissarides (1994). In the model, heterogenous firms and executives form matches to produce output using their combined resources, managerial talent and firm market value. The long-term dynamics of executive compensation is explained by the process of search and matching, including CEO poaching and counter-offers, and the productivity shocks that affect firms. The model is estimated by the simulated method of moments using data from the Compustat and ExecuComp databases.

We show how matches between firms and CEOs can remain viable in the dynamic environment under appropriate contract renegotiations, and generate testable predictions

[1]Data sources: executive compensation data obtained from Execucomp survey; income quantiles are computed based on the methodology by E. Saez, http://elsa.berkeley.edu/ saez/TabFig2018prel.xls; income and earnings data are from the US Census Bureau and The Current Employment Statistics (CES).

about executive compensation and turnover. The estimated cost of mismatch between firms and executives is high, resulting in welfare losses of 40% relative to the frictionless benchmark. The structural framework gives us a robust tool to perform a wide range of counterfactuals related to the dynamic connection between firm value and CEO pay, which can inform corporate finance and policy. We focus on three sources of executive compensation growth: headhunting, pay for luck and CEO entrenchment.

Executive headhunting is widely considered as one of the leading concerns the boards of directors have when setting the CEO pay. External pressures encourage boards to boost compensation packages, thereby ensuring that their executives are not underpaid relative to the market (Kaplan, 2012). In our model firms can make offers to the CEOs of their competitors. If this happens, the incumbent firm can either counter the offer or let the CEO go and look for a replacement. Wage growth that results from these job-to-job transitions and counteroffers accounts for 25% of the total growth in the steady-state equilibrium.

Pay for luck is a practice to reward CEOs for the circumstances outside of their control, such as idiosyncratic shocks to the firm market value (Bertrand and Mullainathan, 2001). We model pay for luck by introducing asymmetric contract renegotiations for good and bad shocks to the firm market value. The CEO contract can only deteriorate if match is no longer feasible for the new market value and previously agreed compensation conditions. However in case of good shocks CEOs have an ability to extract and retain additional share of match surplus, which we attribute to pay for luck. For the estimated values of structural parameters pay for luck generates the remaining 75% of the total executive compensation growth that is not due to headhunting.

The exact extent of pay for luck effect depends on the relative bargaining power of the CEO over the board of directors. We estimate that the bargaining power of incumbent CEOs is almost 40% higher than that of the candidates during initial contract negotiations. We attribute this result to the CEO entrenchment, defined in the literature as the ability to obtain high pay while facing a low risk of job loss. The problem is rooted in the

complicated relationships between CEOs and boards of directors. Although theoretically the board of directors is supposed to make independent decisions on the CEO compensation, in practice CEOs can often influence the choice of directors and their salaries. The board members often serve as CEOs at other companies and follow similar reasoning to set their own salaries, complicating the agency problem (Bebchuk and Fried, 2004). In a counterfactual that sets bargaining power of the incumbent CEOs at its starting level the CEO compensation growth rates are cut by one fourth, an effect approximately equivalent to that of headhunting.

Our results therefore identify the two interrelated problems of pay for luck and CEO entrenchment as the main drivers of executive compensation growth. Policy measures that limit the bargaining power of incumbent CEOs have a considerable potential to control the growth of executive compensation. These could include "say on pay" policies that give firm's shareholders the right of non-binding vote on increase or decrease of executive compensation, pay caps and progressive tax rates at the very top of the distribution.

This paper builds upon a large body of literature on executive compensation; Edmans and Gabaix (2016), Edmans et al. (2017) and Murphy (2013) provide comprehensive reviews of the relevant theoretical and empirical research. We contribute to this literature in two important ways. First, we develop the first dynamic search and matching model of executive compensation with frictions and production complementarities. Second, based on this theoretical framework we provide a quantitative explanation of the main patterns and practices of executive compensation, including the rise in the level and dispersion of executive pay, labor market frictions and peer pressure in the determination of executive pay. To this end, we estimate the model that jointly explains the level of executive pay, its dynamics, and CEO job mobility.

Our model reinforces the main results of the current literature on the level of the CEO pay, which predominantly stem from the assignment models. First, we confirm that equilibrium involves positive assortative matching. Large firms attract and retain the best executives, and the CEO pay increases in firm size as previously shown by Lucas (1978),

Gabaix and Landier (2008) and Tervio (2008). Second, one of the most robust findings in the existing research on executive compensation is that CEO pay increases in the market value of the companies they manage. Gabaix and Landier (2008) show based on the frictionless assignment model of executive compensation that an increase in median firm size will lead to a proportional increase in compensation. As the top corporations keep on expanding through acquisitions and consolidation processes and their operations become more complex, the compensation packages of their CEOs improve as well. In our model with frictions the median executive compensation increases as firms grow, however the firm growth observed over the last twenty years will only explain one third of executive compensation increase. The search frictions and executive compensation practices help to explain the remaining part.

We also contribute to the literature on search and matching models. While there is little research on theoretical modeling of CEO-firm mismatches, such models have been used extensively to study frictions in the general labor markets. Our model is closely related to Lise et al. (2016), which we reinterpret and augment to account for the specifics of the CEO market. The main novel model features introduced in our paper are the start up payments and surplus sharing rules that reflect executive compensation practices.

The remainder of the paper is structured in the following way. Section 2 reviews our data collection and construction. Section 3 outlines the model. Sections 4 and 5 discuss identification, the main estimation results and counterfactuals. Section 6 concludes.

2 Data

Our empirical analysis is based on the ExecuComp and Compustat datasets that contain the key performance indicators of the largest publicly traded firms in the United States and the pay histories of their top executives. ExecuComp is a Standard & Poor's (S&P) database of annual executive compensation extracted from the definitive proxy statements of the public companies. It includes companies that were a part of S&P 500, S&P 400

MidCap or S&P SmallCap 600 indices at least for some of the time since the survey inception in 1992. For each firm in the database, ExecuComp identifies its CEO and four additional executives who received the highest pay in the previous year; it further provides detailed information on their compensation, dates of affiliation with the company and basic demographics. We use a shared firm identifier to link these compensation data to the Compustat dataset containing the main characteristics of the companies.

After accounting for missing data, we obtain a sample of 297,226 executive-year observations. The sample covers 3,741 public companies and 50,232 executives employed by these companies at some period between 1992 and 2017. We identified 8,930 distinct CEO-firm matches that involve 8,243 individuals in the data, including 5,952 completed CEO employment spells. The details of our sample construction and the definitions of the main variables are provided in the data appendix. Table 1 summarizes descriptive statistics. In the rest of this section we document the labor market transitions of the top executives and their wages. These data facts help understand the main features of the market for CEO talent and are essential for our model identification.

We consider executives in the sample as a pool of qualified candidates available to fill the vacant CEO positions. Indeed, 88% of the newly appointed CEOs previously held another senior executive position with a Compustat company. The top jobs are scarce and hard to obtain: only 16% of executives in the dataset ever served as CEOs. Given this interpretation, we define job finding probability as the probability that a non-chief executive would move to a CEO position within one year. The average job finding probability in the data is only 1%. This is very low relative, for example, to the US post-war average annual job finding probability of 90% (Shimer, 2012).

Once obtained, CEO jobs are fairly secure. On average, a CEO stays in the office for a period of 7.2 years, and substantial number of CEO spells end in retirement. Only a half of the CEO appointments end in fewer than five years. For comparison, in the general workforce over 70% of jobs end within five years (US Bureau of Labor Statistics News Release, 2019). Estimated annual probability of job loss, defined as the probability that a CEO

leaves the post within one year, is 14.8%. This is higher than general, although much closer to the long-term national average of 12.9% than the job finding probability.

Low job turnover indicates that top executives rarely change employers among S&P companies. Over the period of observation, executives on average held jobs with 1.1 firms. 91% only worked in top management positions for one employer, although they normally took numerous roles within the same company or its affiliates. Executives with CEO record are slightly more mobile, their average number of employers is 1.3. Among executives who have ever served as CEOs, majority (92%) only did it once, 6.9% twice, and the remaining 1.1% had either three or four CEO jobs. A sizable part of multiple CEO jobs (31%) are due to repeated appointments of the same CEO in one company or its predecessors. Another 17% took place in companies from the same industry defined by four-digit NAICS code. This leaves a half of multiple spells that were recorded in companies unrelated at either organizational or industry level. We conclude that while CEO job-to-job transitions are not frequent, industry specifics do not eliminate the possibility of moves across the entire market.

Multiple CEO appointments help us define and estimate job-to-job transitions. In addition to including direct transitions between CEO jobs, we take into account succession planning practices. It is common for companies to appoint an external replacement as a member of the top executive team ahead of anticipated retirement of the incumbent CEO. In the data this is seen from a significant number of CEOs who joined their companies shortly before taking office: a quarter of the CEOs appeared among the top-five executives for the first time within two years of their appointment. We therefore count all new CEO appointments that happen within two years of finishing the previous CEO spell as job-to-job transitions. Altogether we observe 474 moves that yield an annual probability of job-to-job transition of 1%.

We determine the level and growth rates of CEO pay from ExecuComp variable tdc1, the total annual compensation. This variable aims to reflect a variety of tools and approaches companies use to reward their executives. It includes salary, bonuses,

restricted stock grants, stock options, long-term incentives payouts and any other annual payouts reported in proxy statements. Over the 25 years covered by the data, the average CEO compensation increased by four million US dollars (135%) in real terms. In line with the literature on executive compensation, we find that this change was tallied with a comparable increase in the market value of the firms (Figure 1). In the cross-section, CEOs of larger firms are also consistently paid more (Figure 2). Gabaix and Landier (2008) argue that the rapid growth of executive compensation is largely explained by increasing jobs demands as CEOs have to manage larger and more complicated businesses. Although the growth rate of CEO compensation is positively related to the growth rate of firm market values, the relationship is driven by positive changes. Conditional on positive shock to the firm market value, CEO compensation on average increases by 9% per annum. However the average change of compensation given a negative shock to firm market value is not statistically different from zero. These are essential data fact that are captured by our model.

The top managers of public companies such as CFOs and COOs place in the top percentile of the US income distribution with average annual earnings above three million dollars. Transitions from such jobs to the CEO role are associated with an average compensation increase of 18%. Job-to-job transitions carry a premium in excess of one million dollars, which amounts to 21% of the average CEO compensation and 133% more than the compensation in the previous job. The CEO pay continues to grow with tenure at an average rate of 33% per annum. Alongside job-to-job transitions, we use wage growth with tenure to identify the role of poaching in the setting of executive compensation.

To summarize, the market for executive talent is characterized by extremely high levels of pay and low rates of job finding and job loss. Transitions between jobs are infrequent, but present to the extent suggesting a possibility of credible threat from poachers across the market. Although limited, CEO job mobility appears to exist to the extent enough to put pressure on the boards and claim that compensation practices such as benchmarking and pay for luck may be well grounded in the CEO labor market. Our modeling approach

and identification are informed by these findings.

3 Model

We develop a dynamic equilibrium model of wage determination and employment with heterogenous firms and executives. The model accounts for frictions due to search and mismatch, and generates endogenous job creation and job destruction triggered by productivity shocks as in Mortensen and Pissarides (1994). Empirically determined productive complementarities lead to sorting as in Shimer and Smith (2000). We allow on-the-job search and poaching that cause wage growth in long-term contracts as in Postel-Vinay and Robin (2002). Within search and matching literature our model is most closely related to Lise et al. (2016), which we augment and reinterpret to account for the specifics of the CEO market. We introduce several novel model features that reflect essential executive compensation practices: entrenchment, performance incentives and starter packages. Wage dynamics are explained by the process of search and matching and the productivity shocks that affect the firm. We proceed with a description of the main model elements.

3.1 Firms and managers

The economy is composed of an endogenously determined number of firms N. There is a unit mass of qualified candidates who are eligible to manage these firms as CEO's. Candidates are characterized by permanent differences in their managerial ability or talent x, which is continuously distributed on the support $[x_{min}, x_{max}]$ with density $l(x)$. We assume that managerial ability is universal rather than firm or industry specific. This is consistent with observed mobility of the top executives across firms and industries, and diverse nature of business conducted by many publicly traded companies. We further assume that the candidates already accomplished all their general education and training and do not acquire additional skills during the part of their careers covered by the model,

so that there is no human capital accumulation.

Firms are different in size and the level of production technology that they can access, which is summarized by their market value y continuously distributed on the interval $[y_{min}, y_{max}]$ with known density $n(y)$. We use market value as the measure of firm size following Gabaix and Landier (2008) who compare the total market value, earnings before interest and taxes and sales as possible candidates and choose the market value as their preferred option because of the strength of statistical relationship with CEO compensation.

The market value evolves through persistent idiosyncratic shocks according to a jump process. Shocks to the firm value arrive at rate δ, and whenever this happens a new value $y' \in [y_{min}, y_{max}]$ is drawn from a continuous distribution with density $\gamma(y'|y)$. Parameter δ determines the persistence of shocks: shocks are more persistent when δ is low. Examples of shocks may include shifts in market demand for firm's products, innovations in management practices and technology, changes in financial markets and policy, or releases of relevant news in the media.

The ability of candidates are observed by both parties, but not by the researcher. Market values of the firms are known and observed. Time is continuous. We assume that both firms and candidates are risk neutral and discount future at rate r. Firms are infinitely lived, and executives retire exogenously at rate μ.

3.2 Matching process

A match between an executive and a firm combines managerial talent x with corporate resources y to produce a flow of output $f(x, y)$. A production function $f(\cdot, \cdot)$ that is complementary in talent and firm value would generate positive assortative matching: that is, better CEOs will be matched to firms with higher value. Productive complementarity increases the aggregate output and generates large wage dispersion as found in the CEO data and literature. Importantly, this is not a model assumption like for example in Tervio (2008), as we determine the exact degree of complementarity empirically.

Matches can be destroyed either endogenously or exogenously. Endogenous job destruction is driven by the market value shocks and outside offers. The shocks to the market value can prompt renegotiation of contracts between firms and the CEOs and lead to separations when no new contract can be agreed to keep a match viable. Matches can also dissolve when a CEO receives a new offer that can not be countered by the current firm.

The exogenous rate of job destruction ξ represents separations that happen regardless of the productive characteristics of either the CEO or the firm. We assume that a firm can not unilaterally fire its current CEO and replace her with a new candidate because of binding contractual agreements. It may however renegotiate the CEO's compensation downwards in response to the market value shocks, possibly to the point when the CEO decides to quit. After the CEO quits, the firm survives and stays open until a new match is found. Following the literature, we assume free entry of vacancies.

In the sequel, we use the following notational convention. Given the number of candidates belonging to each type x in the economy $l(x)$, define the endogenous number of candidates of this type who do not hold executive positions as $u(x)$. Further, let the number of firms with market value y in the economy be $n(y)$ and the number of firms with vacant CEO positions $v(y)$. The number of free candidates available to take the CEO positions is then computed endogenously as $U = \int u(x)dx$, the number of executive jobs in the economy as $N = \int n(y)dy$, and the number of open positions as $V = \int v(y)dy$. The distribution of matches in equilibrium is described by density $h(x, y)$.

The labor market frictions are modeled using a standard matching function $M(\cdot, \cdot)$ that takes as its arguments the number of efficiency units of the eligible candidates $U + s(1 - U)$ and the number of open CEO positions V. Parameter s is the relative search intensity among the incumbent CEOs as compared to candidates without executive jobs. The matching function determines the number of meetings between firms and candidates per unit of time given supply and demand of jobs. It is concave, increases in both arguments and exhibits constant returns to scale. Using the notation above, all four

relevant meeting rates can be computed with parameter

$$\kappa = \frac{M(U + s(1-U), V)}{[U + s(1-U)]V}. \tag{1}$$

The rate at which candidates meet vacancies of type y is $\kappa v(y)$ for free candidates and $s\kappa v(y)$ for the incumbent CEOs. Vacancies meet free candidates at rate $\kappa u(x)$, and incumbent CEO's at rate $s\kappa h(x, y)$.

3.3 Wages

We start this section with the definition of the outside options of the firms and candidates. We then describe the wage setting mechanism that firms and CEOs employ in order to split surplus generated by a match. The wage setting mechanisms involves three elements that determine respectively the wages offered to new CEOs, the wages offered to the CEOs attracted via headhunting, and the wages renegotiated over the course of the contract as a result of the market value shocks. Wages are determined by bargaining over surplus in case of new hires and changes to firm productivity, combined with Bertrand competition over workers in case of headhunting.

An unmatched candidate without a current executive position receives a flow income $b(x)$ that depends on her ability and can be interpreted as income from non-executive jobs and related activities. The value of unmatched state for a candidate x is denoted by $W_0(x)$. It is comprised of the non-CEO income flow $b(x)$ and the expected present value of the income to be received if this candidate took a CEO job. The value of unmatched state for a firm y that has a vacant CEO position is given by $\Pi_0(y)$ and involves the expected returns from hiring a new CEO net of the search cost.

The value of matched state for a candidate with ability x who is employed by a firm with market value y and is paid a current wage w is denoted by $W_1(x, y, w)$ and reflects the present value of the CEO compensation contract. The corresponding state value for the firm, $\Pi_1(x, y, w)$, reflects the present value of profit. As in Mortensen and Pissarides

(1994), the surplus generated by the match, $S(x, y)$, is defined as the sum of surpluses received by the CEO and the firm:

$$S(x, y) = \Pi_1(x, y, w) - \Pi_0(y) + W_1(x, y, w) - W_0(x). \tag{2}$$

A match is feasible and sustainable if $S(x, y) \geq 0$. We assume that there exists at least one feasible match for each ability level x, so that each candidate potentially qualifies for a CEO position in some firm. Match formation is efficient, which implies that a match is formed whenever a firm meets a candidate such that they have a potential to create positive surplus. We now turn to the three elements of the wage determination process.

1. New hires

When a firm hires a new executive from the pool of free candidates, the contract between the two is determined by Nash bargaining. The CEO's initial bargaining power is determined by parameter β_0, which we estimate from the data. The match surplus is split between the firm and its new CEO, so that the wage w solves

$$W_1(x, y, w) - W_0(x) = \beta_0 S(x, y). \tag{3}$$

2. Renegotiations due to market value shocks

The initial contract only remains viable if it can compensate both parties for the value of their outside options. The match surplus and the values of outside options change with the arrival of shocks to the firm market value, and these changes may trigger contract renegotiation. A viable match renegotiates whenever either side has an incentive to separate unless offered a better deal. In addition, CEOs are able to use their bargaining power and appropriate a fraction of any extra surplus created as a result of productivity shock. The latter feature captures asymmetry in the response of CEO compensation to good

and bad market developments. For example, Taylor (2013) documents an asymmetric response of compensation to good versus bad news about CEO ability. In his model, the average CEO captures roughly half of the surplus from good news, but none of the negative surplus from bad news. Similarly in our model CEOs always harvest some of the gains from good shocks and are only affected by bad ones in sufficiently severe situations.

The renegotiation mechanism works as follows. Suppose a shock shifts the firm market value from y to y', changing the match surplus from $S(x, y)$ to $S(x, y')$. The contract may be renegotiated in the interest of both parties as long as there is enough surplus to provide proper incentives. The following five cases are possible:

1. A shock made the match surplus negative, $S(x, y') < 0$. The match is no longer viable and is endogenously destroyed. The CEO leaves the job and returns to the pool of candidates. The firm looks for a replacement.

2. The new surplus is non-negative but the CEO's agreed share exceeds the total surplus,

$$0 \leq S(x, y') < W_1(x, y', w) - W_0(x).$$

   The contract has to be renegotiated because firm can no longer afford paying the old wage. Under the new contract the CEO receives the entire net surplus, $W_1(x, y', w') = W_0(x) + S(x, y')$. We assume that firms do not search to replace their active CEOs, but may renegotiate wages down following a productivity shock if, for example, the firm's outside option value changed as a result of such shock.

3. The new surplus is non-negative but lower than the original surplus, $0 \leq S(x, y') < S(x, y)$, and the CEO's outside option value exceeds the current wage,

$$W_1(x, y', w) - W_0(x) < 0.$$

   The CEO, who now finds the outside option more attractive, would leave unless the firm makes a better offer. The new wage is set so that to make the CEO indifferent

between the current job and the outside option, $W_1(x, y', w') = W_0(x)$.

4. The shock increased match surplus, $S(x, y') \geq S(x, y)$. The wage contract is renegotiated to give the CEO at least a fraction β_1 of the new surplus without losing any previously accumulated wage gains. The new wage is determined by

$$W_1(x, y', w') = \max\{W_1(x, y', w), W_0(x) + \beta_1 S(x, y')\}.$$

5. The surplus remains non-negative following a shock and does not increase, the adjusted value of the job for the CEO is above the value of the outside option, and the firm can afford paying previously agreed compensation,

$$0 \leq W_1(x, y', w) - W_0(x) \leq S(x, y').$$

In this case the current contract remains unchanged, the CEO continues work with contract $W_1(x, y', w)$ and old wage w.

Renegotiation of wage contracts is asymmetric with respect to good and bad shocks. When renegotiating following a bad shock, we look for the smallest possible adjustment to the original contract that would make the match viable. There is no Nash bargaining, and the full surplus is given to the CEO. This mechanism is useful because it preserves the gains accumulated by the CEOs from earlier counter-offers. The party that gets full power in wage negotiation depends on the value of outside options: whoever would be better off outside the match based on the comparison between non-executive employment and unfilled CEO position values gets the first hand.

The model generates negative correlation between the volatility of firm market values and the level of CEO compensation. This reflects the trade-off between risk and executive compensation, as proposed by Aggarwal and Samwick (1999) who document strong negative association between pay-performance sensitivities and stock return variance.

Wages can only decrease when the value of the CEO contract exceeds the current

surplus. However CEOs also have an ability to extract a fraction of any increases in surplus according to their bargaining power, so wages can grow preserving any gains that accumulated through headhunting whenever the match surplus improves. This wage determination mechanism captures the fact that CEO compensation tends to grow more when firms perform well while staying resilient in face of bad news, which we documented in Section 2.

Importantly, the direction of the relationship between productivity shocks and wages is not obvious. An adverse shock may nevertheless improve the CEO's contract, and a positive shock may result in a lower wage as the firm may become better off finding a new CEO.

Although non-vacant firms cannot actively search for a CEO replacement, they can fire a CEO when the current surplus becomes too low. In the model this happens as a result of shocks to the firm value. One possible interpretation is poor performance of the executive, as market values may fall because of bad business decisions. A firm can then start to be on the lookout for a replacement.

3. External poaching

A firm may approach a candidate currently working for another firm. In this case, incumbent firm responds to the outside offer, and a negotiation game is played between the CEO and two firms as in Dey and Flinn (2005) and Cahuc et al. (2006). Suppose that a CEO who is currently in a match (x, y) receives an offer from a firm y'. The following three cases are possible.

1. The match with new firm will create higher surplus, $S(x, y') > S(x, y)$. In this case the CEO accepts a new offer and moves. The new wage w' is set to give CEO the entire surplus of the old match, $W_1(x, y, w') - W_0(x) = S(x, y)$. As with all new appointments, the The CEO also receives a lump sum starting incentive package $w_0(x, y)$. This setup is somewhat similar to headhunting in Postel-Vinay and Turon (2010), except that the CEO does not get a fraction β_0 of the difference in two

surpluses in addition to the surplus of the old match. Hence, in our model less wage growth is generated by counteroffers than by changes in the firm values, which is consistent with the data.

2. The surplus that can be created at the new firm is lower than that at the old firm, but higher than the CEO's current share of surplus:

$$W_1(x, y, w) - W_0(x) < S(x, y') < S(x, y).$$

In this case the CEO stays with the old firm and uses the outside offer to improve the contract. The new wage is set to give CEO the entire potential surplus of the new match, $W_1(x, y, w') - W_0(x) = S(x, y')$.

3. The new match cannot yield higher surplus and does not exceed the share of surplus that the CEO receives under the current contract,

$$S(x, y') \leq W_1(x, y, w) - W_0(x) \leq S(x, y).$$

There is no credible threat and wage cannot be renegotiated, or possibly the CEO already extracted all surplus from the current match as a result of earlier negotiations. The CEO stays with the current firm and receives the same wage.

3.4 Value functions

The following value functions determine decision rules for the CEO's and firms, formalizing earlier discussion of the environment and decision rules. Technical appendix contains full details of the relevant derivations.

1. CEO candidates without jobs

For a candidate with talent x, the present value of being in the pool of candidates without

a CEO job is defined by

$$(r+\mu)W_0(x) = b(x) + \kappa\beta_0 \int\limits_{y'\in\mathcal{A}(x)} S(x,y')v(y')dy', \tag{4}$$

where $\mathcal{A}(x) = \{y : S(x,y) \geq 0\}$ is a set of firms that can form a sustainable match with the candidate of type x. The first additive term, $b(x)$, represents income flow associated with staying in the pool of candidates. The second term accounts for the expected present value of the possibility to meet and accept a CEO appointment. Candidates meet vacancies of type y' at rate $\kappa v(y')$, and accept offers only if proposed match can generate positive surplus ($S(x,y') > 0$). If an offer is accepted, the candidate will receive fraction β_0 of surplus. Integration is over all firm types y' that could potentially approach a candidate.

2. Firms with vacant CEO positions

Firms with vacant CEO positions have to pay search cost c until a position is filled. The new CEO may either come out of the pool of candidates or be headhunted from another firm. In the meantime, firm's market value continues to evolve under the influence of exogenous shocks. These conditions determine four additive terms in the option value equation for firm's profits,

$$\begin{aligned} r\Pi_0(y) = -c + \delta \int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' \\ +\kappa(1-\beta_0) \int\limits_{x'\in\mathcal{B}(y)} S(x',y)u(x')dx' \\ +s\kappa \iint\limits_{(x',y')\in\mathcal{C}(y)} \big[S(x',y) - S(x',y')\big]h(x',y')dx'dy', \end{aligned} \tag{5}$$

The first term in the equation above represents the cost of keeping a vacancy open. The second incorporates the effects of future market value shocks: shocks arrive at rate δ, and change the firm's market value from y to y' and the present value of keeping a firm

with unoccupied CEO position from $\Pi_0(y)$ to $\Pi_0(y')$. Integration is over the distribution of market values $\gamma(y)$. The third term represents the expected gain from matching with unemployed candidate. Firms meet jobless candidates at the rate $\kappa u(x')$, employ them if a positive surplus can be created, and receive their share $1-\beta_0$. Integration is over the set of sustainable matches $\mathcal{B}(y) = \{x' : S(x', y) \geq 0\}$. The final term represents expected gains from headhunting: $s\kappa h(x', y')$ is the rate at which a firm makes contact with employed CEOs and match can only be formed if the pair can produce surplus higher than the surplus of the existing match. In case of success, the firm will retain all additionally generated surplus. Integration is over the set of sustainable matches $\mathcal{C}(y) = \{(x', y') : S(x', y) \geq S(x', y')\}$.

3. Employed CEOs

The present value of the current contract for a CEO of type x employed by firm y that pays wage w is determined from

$$\begin{aligned}
&\left[r+\mu+\delta+\xi+s\kappa\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy'\right][W_1(w,x,y)-W_0(x)] \qquad (6)\\
&\qquad = w-b(x)-\kappa\beta_0\int_{y'} S(x,y')^+ v(y')dy'\\
&+\delta\Bigg[\int_{y':0\leq S'<\Delta W'} S(x,y')\gamma(y')dy' + \int_{y':S\leq S',\Delta W'<\beta_1 S'} \beta_1 S(x,y')\gamma(y')dy'\\
&\qquad + \int_{y':\beta_1 S'\mathbb{1}\{S\leq S'\}\leq\Delta W'\leq S'} [W_1(w,x,y')-W_0(x)]\gamma(y')dy'\Bigg]\\
&+s\kappa\int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x,y),S(x,y')\}+w_0(x,y')\mathbb{1}\{S(x,y)\leq S(x,y')\}\big]v(y')dy',
\end{aligned}$$

where the set of market values $\mathcal{A}(x,y,w) = \{y' : W_1(x,y,w) - W_0(x) < S(x,y')\}$ defines poaching firms that can generate a positive change to the CEOs wage and $\int_{y'\in\mathcal{A}(x,y,w)} v(y')dy'$ gives the total number of such firms in the equilibrium. The firm starts by paying wage w, then wage can be renegotiated, job can be destroyed, or the CEO can move to a different firm. We omit the present value of the firm with an appointed CEO because

is not involved in the problem solution, but it is straightforward to compute in similar way.

4. Match surplus

The three value functions for unemployed CEO, vacant firm and a CEO employed with wage contract w allow to express the match surplus $S(x, y)$ so that it does not depend on the current wage contract. The match surplus is defined by the fixed point of the equation:

$$\begin{aligned}(r+\xi+\delta+\mu)S(x,y) = f(x,y) - b(x) - \kappa\beta_0 \int\limits_{y'\in\mathcal{A}(x)} S(x,y')v(y')dy' \\ +c - \kappa(1-\beta_0)\int\limits_{x'\in\mathcal{B}(y)} \big[S(x',y) - w_0(x',y)\big]u(x')dx' \\ -s\kappa \iint\limits_{(x',y')\in\mathcal{C}(y)} \big[S(x',y) - S(x',y')\big]h(x',y')dx'dy'.\end{aligned} \tag{7}$$

That is, the discounted surplus is expressed in terms of the current output of a match $f(x, y)$ net of the worker opportunity cost (terms 2 and 3) and firm opportunity cost (terms 4 and 5), the expected change of surplus due to counteroffers, and the expected change of surplus due to productivity shock (term 6). The proof of this result is based on the equation for the present value of match output $P(x, y) = S(x, y) + W_0(x) + \Pi_0(y)$ and is derived in the appendix.

3.5 Steady-state equilibrium

Equilibrium is efficient, so that in equilibrium all agents follow their optimal strategies. The four equilibrium objects that are endogenously generated by the model are the distribution of matches $h(x, y)$, the number of firms in the economy N, market tightness $\kappa(u, v)$ and surplus $S(x, y)$. Additionally, the distribution of candidates and vacancies $u(x)$ and $v(y)$ are generated and summarized in the market tightness. The remaining elements of the model are determined exogenously and have to be either assumed or

estimated.

The distribution of matches in equilibrium is described by the density $h(x, y)$ which satisfies balance conditions for each ability level x and market value y:

$$\int h(x, y)dy + u(x) = l(x) \tag{8}$$

and

$$\int h(x, y)dx + v(y) = n(y). \tag{9}$$

That is, for each type x, the total number of candidates is split between CEO's and free candidates, and for each type y, the total number of positions is split between occupied and vacant.

The equilibrium distribution of matches $h(x, y)$ is determined from the steady state flow expression that equates flows into and out of the candidate pool. The outflows are generated by exogenous job destruction, job loss caused by productivity shocks, poaching that makes CEOs switch firms, and mortality. Taking account of the relevant arrival rates, the outflows are given by

$$E_{out}(x, y) = h(x, y)\big[\xi + \delta + \mu + s\kappa \int\limits_{y' \in \bar{\mathcal{B}}(x,y)} v(y')dy'\big],$$

where $\bar{\mathcal{B}}(x, y)) = \{y' : 0 \leq S(x, y) \leq S(x, y')\}$ is a set of potential surplus improving matches.

The inflows are due to meetings between candidates and incumbent CEOs with unmatched firms, plus transitions of firms to type y due to productivity shocks:

$$E_{in}(x, y) = \delta \int h(x, y')\gamma(y|y')dy' + \kappa\nu(y)\big[u(x) + s \int\limits_{y' \in \mathcal{B}(x,y)} h(x, y')dy'\big],$$

where $\mathcal{B}(x, y)) = \{y' : 0 \leq S(x, y') \leq S(x, y)\}$ is a set of matches that do not generate surplus improvement.

In a stationary equilibrium, the flows in and out of the candidate pool must be balanced. This implies that for all feasible matches (x, y) such that $S(x, y) > 0$ the following condition has to be satisfied:

$$h(x,y)\big[\xi + \delta + \mu + s\kappa \int\limits_{y' \in \bar{\mathcal{B}}(x,y)} v(y')dy'\big] \tag{10}$$
$$= \delta \int h(x,y')\gamma(y|y')dy' + \kappa\nu(y)\big[u(x) + s \int\limits_{y' \in \mathcal{B}(x,y)} h(x,y')dy'\big].$$

Equations (10), (8) and (9) define the equilibrium distribution of matches $h(x, y)$.

Finally, we determine the equilibrium number of firms N. The state value for vacant firm with lowest market value is zero, $\Pi_0(y_{min}) = 0$. Substitute this condition into the value function (5) and find the equilibrium number of firms N from

$$c = \delta \int \Pi_0(y')\gamma(y')dy' \tag{11}$$
$$+\kappa(1-\beta)\int S(x',y)^+ u(x')dx' + s\kappa \int\int [S(x',y) - S(x',y')]^+ h(x',y')dx'dy'.$$

In equilibrium with infinitely lived jobs the number of type y jobs is given by $n(y) = N\gamma(y)$. This completes our model description.

4 Estimation and identification

For a given set of model parameters, the equilibrium objects are computed in three stages by a fixed point iterative algorithm. We provide details of this procedure in Appendix A.3. The model solution yields the value of job $W_1(x, y, w)$ and the value of the outside option $W_0(x)$, which we use to simulate a panel of CEO employment histories. We then use the method of simulated moments to find parameters that provide the best data fit. We invert the variances of the data moments and use them as weights in the criterion function. We minimize the criterion function by Markov Chain Monte Carlo (MCMC) method, following the algorithm of Chernozhukov and Hong (2003).

4.1 Parametric specification

To complete the model, we require several assumptions regarding some of its elements and their parametric specification. These include the functional forms of the distributions of firm and CEO types $l(x)$ and $h(y)$, the form of production and matching functions, $M(x, y)$ and $f(x, y)$, the distribution of productivity shocks $\gamma(y'|y)$, and the discount rate r.

We assume that market values of the firms and the CEO talent each follow a truncated lognormal distribution. The lognormal distribution is widely used to model firm values and is known to fit the data well , it is also used in the models of competition for talent and superstar wages . The four parameters that define each distribution (mean, standard deviation and truncation points) are estimated alongside the other structural parameters.

We assume that the match production function $f(x, y)$ is CES in the CEO and firm characteristics, with scale parameter A, weight on firm market value relative to the CEO talent $\alpha \in [0, 1]$, and complementarity parameter ρ:

$$f(x, y) = A\big[\alpha x^{\rho} + (1 - \alpha)y^{\rho}\big]^{1/\rho}. \tag{12}$$

Sorting in the model is linked to the complementarity between firm value and CEO talent in the match production function. Parameter ρ determines the modularity of the production function and degree of sorting in the model. Positive sorting is associated with the values $\rho < 1$, and the degree of sorting is stronger for lower values. In the extreme case $\rho = 1$ there is no gain from sorting. The CES specification also allows for the "span of control" issue: it is more difficult to manage big firms, which requires diminishing returns to scale.

The matching function is Cobb-Douglas with constant returns to scale,

$$M\big(U + s(1 - U), V\big) = \eta\big[U + s(1 - U)\big]^{\gamma} V^{1-\gamma},$$

which is a standard specification in the literature . We set the matching elasticity parameter $\gamma = 0.5$ and estimate matching efficiency parameter η and relative search intensity s alongside other structural parameters of the model. The chosen value of the matching elasticity parameter is in the middle of the range of recent estimates used by matching literature (see discussion in Borowczyk-Martins et al. (2013)).

The distribution of shocks to the market value γ and the arrival rate of productivity shocks delta δ are determined empirically from the dynamics of firm market values. The value of the outside option $b(x)$ and the vacancy cost $c(y)$ are normalized to be proportional to the median output flow. We set the annual discount rate to $r = 0.05$. The unexplained variation in annual wages is assumed to be a normally distributed i.i.d. error, with mean zero and estimated variance σ_e^2.

4.2 Identification

We estimate 21 structural parameters of the model by the method of simulated moments (MSM) which minimizes the distance between the sample moments and their equivalents in the simulated dataset. We exploit 261 data moments that capture the frequency of transitions in, out, and between CEO positions alongside the means, variances and growth rates of wages associated with these transitions. The labor force transition moments are computed by year in the labor force, and wages by tenure over a ten year period. We also match the average length of executive lifetime and the mean of non-CEO wages. On the firm side we include the mean, variance, minimum and maximum of the sample distribution of firm market values and the correlation between the market value of a firm and the compensation of its chief executive.

Table 3 summarizes the fit of time-invariant moments which are all matched very closely. In addition, Figures 3 and 4 plot simulations against data moments of the annual labor force transitions and wages. The model captures the main patterns in the data, qualitatively as well as quantitatively. While most of the model parameters are simultaneously connected to several simulated moments, we are always able to pin down a set

of moments that are especially sensitive to changes in a particular estimated parameter. The identification argument unfolds in the following manner and shows that our choice of moments is sufficient for identification of the model.

Having relevant information on firm characteristics, as well as the specific nature of the relationship between CEOs and firms that assume a one-to-one match, gives us a clear advantage in identification of the distribution of firm productivity x, which otherwise often poses problems in the search and matching literature. We use the mean, variance, minimum and maximum values of the firm market values to identify corresponding parameters of truncated lognormal distribution (μ_y, σ_y, y_{min} and y_{max}). The distribution of CEO talent is characterized by the mean μ_x, standard deviation σ_x and lower and upper truncation points x_{min} and x_{max}. These are identified by the corresponding sample moments of the wage distribution: sample mean, standard deviation, and the lower and upper 1% quantiles.

The labor market transitions recorded in the data identify parameters that determine CEO mobility between the labor force states. Matching efficiency η is identified by the annual probability of transition from the pool of candidates to a CEO position. The relative search intensity among incumbent CEOs, s, is identified by the frequency of transitions between CEO positions in different firms. These two parameters determine the job finding rates among candidates and incumbent CEOs.

The overall rate of job loss depends on three model features: voluntary separations caused by productivity shocks, exogenous job destruction and mortality. Mortality rate is directly linked to the length of executive lifespan in the data. We use the mean number of years executives are observed in the data to pin down the rate μ at which CEOs and candidates drop out due to retirement and mortality. Productivity shocks arrive at rate δ which is related to the annual probability of job loss and the growth rates of wages within CEO employment spells. We identify parameter δ using time variation of the market values within firms. The exogenous match destruction rate ξ captures residual separations that are not accounted for by either productivity shocks or mortality and is

identified by transitions from CEO jobs into the candidates pool.

The initial bargaining power parameter β_0 is identified by the mean starting wage of the first-time CEOs and subsequent rate of wage growth associated with job-to-job transitions. When the CEO bargaining power is lower, initial appointments allocate a small fraction of surplus to the CEO with a large potential for future growth due to counteroffers and productivity shocks. When the bargaining power is high, CEO extracts most of the surplus immediately upon the first match formation thus limiting possibilities of future wage growth. The bargaining power of incumbent CEOs, β_1, is identified by rate of wage growth within employment spells. The value of outside option b is pinned down by the average wage of executives who do not hold a CEO post.

Parameters of the production function are identified by wages and relationship between wages and firm market values. The weight on the CEO input into production function α is identified by the regression coefficient from the relationship between CEO wages and market values of the of employing firms. This is because α plays a crucial role in shaping the pattern of the best matches that can be formed between various types of workers and firms. The relatively high correlation between wages and market values is achieved when the matrix of surplus values $S(x, y)$ contains the highest values around the second diagonal, so that matches between the top and bottom types are the least profitable. The complementarity parameter ρ determines the degree of sorting in the model. Complementarity determines potential gains from hiring a better CEO, so that higher degree of complementarity makes more productive firms pay higher premium when poaching a CEO from competitor. We identify ρ from the relative wage growth and wage variance over time within and between jobs. We can not easily identify separately the scaling parameter A and the mean of the CEO talent distribution μ_x which are both primarily related to the mean level of wages. This however does not affect our results as long as we can identify the other parameters of the production function, which we do as described above.

The cost of keeping an open vacancy c justifies whether it is profitable for a firm

without a CEO to stay open, and therefore regulates the number of vacancies in the market. In the absence of reliable information on the CEO vacancies we identify c by the ratio of new appointments made over a year to the number of candidates on the market. The general idea behind this proxy moment is that the number of vacant firms is expected to be very low relative to the number of available candidates. The variance of the measurement error captures the residual wage variance that is not generated by the model.

5 Estimation results

The estimates of model parameters and their standard errors are summarized in Table 2. The model provides a very close fit to all of the key time invariant moments, as shown in Table 3. Figures 3 and 4 provide further illustration for the quality of fit by time. Figure 3 shows the labor force transition moments, which all match the main qualitative patterns in the data. Figure 4 illustrates the dynamics of wage related moments. In the next section we discuss individual estimates, with the main emphasis on the parameters that have important economic interpretation. Next we discuss several results that are consistent with those of assignment models of executive compensation. We finish this section with a discussion of counterfactuals and an evaluation of the role that search friction play in CEO wage determination.

5.1 Estimates of the structural parameters

Estimated parameters of the model reflect the specifics of executive labor market where only a very limited number of vacancies is available to a large number of interested candidates. The value of matching efficiency parameter, $\eta = 0.041$, suggests high level of friction and mismatch. For comparison, matching efficiency estimated both for the entire US economy and by industry all yield values in excess of 0.3 (Sahin et al., 2014, Veracierto, 2011). Consistently with the data, the model predicts that only 11% of the

qualified candidates would be able to hold an office at any point of time. Given the market tightness of 0.008, congestion is so high that on average 124 candidates would be competing for a single vacancy. This is drastically different from the general labor market where employment rate commonly exceeds 90% and the mean market tightness estimated from the Job Openings and Labor Turnover Survey (JOLTS) dataset is about 0.5 (Chéron and Decreuse, 2017).

This particular structure of executive labor market is perhaps best understood from inspection of the relevant probabilities of contact and match formation. The probability of contact between candidates and firms depends on the matching efficiency η, and it comes as no surprise that the coveted chief executive jobs are extremely hard to find. A qualified top manager has once in a lifetime opportunity to meet a vacant firm, which on average only happens every 25 years. The situation is entirely different for vacant firms which meet candidates approximately 5 times a year. Conditional on contact being made, matches are consummated at a rate of 85%. This happens whenever a match has potential to generate nonnegative surplus, which in equilibrium is the case for 89% of all possible matches.

The relative positions of firms and executives are quite different in the poaching submarket. With search intensity parameter $s = 1.36$, incumbent CEOs enjoy significant advantage in their access to new jobs over non-employed candidates. In the executive world this is likely to be a product of higher visibility, publicity and outstanding networking opportunities that are intrinsic to the top appointments. On average incumbents receive offers from poachers once every 18 years, a gain of almost one third over non-CEO contestants. In the meantime low number of jobs makes it harder for poachers to contact CEOs of their competitors: on average they succeed once in 0.9 years. Due to the differences in firm productivities, only 42% of the meetings between vacant firms and incumbent CEOs result in new matches. A further 23% lead to wage renegotiations with the current employer and the rest represent unsubstantiated challenges that did not lead to contract changes. Because employment does not constraint search,

candidates derive no gain from rejecting an offer in anticipation of a better one. This outcome marks another difference from the general labor market where empirical evidence suggests lower search intensity and job arrival rate among employed individuals (Postel-Vinay and Robin, 2002, Faberman and Kudlyak, 2019).

Although making contacts with firms is very challenging for candidates, they enjoy substantial bargaining power in the initial contract negotiations and manage to extract 50% of the match surplus. Once appointed, CEOs reinforce their positions within companies and increase their share of surplus gains resulting from changes in firm market values to 69%. The starting bargaining power determined by parameter β_0 is quite high yet leaves sufficient room to sustain subsequent wage growth due to counteroffers. The wage growth generated by the model arises from the job search process and improvement in sorting over time, as well as by changes in firm productivity and appropriation of additional surplus determined by β_1. We decompose and analyze the relative importance of these two channels for the growth of executive compensation the next section.

Parameters μ, ξ and δ determine the frequency of separations associated respectively with retirement, exogenous shocks to the match and changes to the firm market value. Retirement is by far the most common reason for CEOs to leave their jobs; it explains 57% of all separations. The estimate of retirement rate μ implies an average executive career length of 9 years. Exogenous layoffs are relatively rare events that happen approximately once every 12 years, yet in the environment with low CEO turnover they accumulate to account for further 37% of separations. Although market value shocks occur substantially more often, on average every 1 years, they only generate 6% of recorded separation. CEO appointments are resilient to market value shocks because majority of matches are capable of producing positive surplus: conditional on receiving a productivity shock, the match survives with probability of 0.916. As we discussed in the data section, Execucomp is not very effective in recording precise reasons of CEO separations and does not gives us a reliable benchmark, however based on our in-depth pilot study of CEO career pathways we think that the model gives a plausible description of this market.

The value of an outside option for free candidates and the cost of maintaining a vacancy for unmatched firms are both normalized relative to the match output. A candidate's flow income is equivalent to 27% of output that would be produced if they were matched to a firm of average market value. Vacancy cost is estimated to be 6 times higher than the flow output of a firm when matched to an average candidate. High estimate of the cost of CEO replacement is not unique to our paper. Taylor (2010) examines CEO turnover by constructing and estimating a dynamic employment model, where a board maximizes shareholder value. The board decides whether to fire the CEO each year, depending on the imputed CEO skill. To fit the turnover rate, the model requires the average board to behave as if replacing the CEO costs shareholders at least $200 million. Taylor indicates that this cost largely reflects entrenchment, which also seems to be the driver behind our estimate.

One of the main features of executive compensation documented by the literature is that in a cross-section CEO pay is proportional to a power function of firm's size. Our estimates are consistent with this fact, which Edmans and Gabaix (2016) refer to as Roberts's Law. The value of pay-firm size elasticity in our model is identified from the regression of log wages on log market as $\beta_{xy} = 0.307$. This is very close to the data moment of 0.285, and just slightly below the pay-firm size elasticity of around one third that is predicted by the power models.

The estimates of production function parameters show that matches place a weight of $\alpha = 0.69$ on the CEO talent. Degree of complementarity between the value of a firm and the talent of its CEO is determined by parameter $\rho =$ -0.806, which implies the elasticity of substitution between managerial talent and market value of 0.554. This indicates complementarity between managerial talent and firm values, and substantial degree of sorting between firms and managers. We discuss the implications of this estimate in the next section, and compare our results on sorting to those of well established assignment models.

5.2 Match surplus and assortative matching

Equilibrium involves positive assortative matching, as seen from Figure 5 that shows equilibrium distribution of matches. In case of perfect sorting, matches would fall onto the diagonal line shown in red on the graph. Given frictions, sorting is in the model is far from perfect yet the distribution of matches clearly gravitates towards the diagonal. Our sorting results are consistent with the main predictions of assignment models of executive compensation, such as the equilibrium assignment model of CEO pay of Gabaix and Landier (2008).

The distribution of matches and sorting are driven by the shape of surplus function $S(x, y)$ plotted in Figure 6. The highest surplus is produced by a match between the largest firm and the most skillful candidate. At the top of the distribution both parties have strong incentives to look for a match with the best counterpart available. For the best executive a move from the job in a median firm to the top one is associated with a match surplus gain of 279%. Firms benefit even more from finding the right match: the best firm managed by a CEO in the 90th ability quantile could generate 1.8 times higher surplus if it replaced its chief executive with the most capable one.

Not all matches are viable: surplus is negative for 11% of all possible firm-executive pairs which correspond to white areas in Figure 5. Production complementarities generate minimal thresholds for the ability and market value inputs required in order for a match to generate non-negative surplus. Infeasible matches occur at the corners of the surplus surface: the worst firms do not match with the best CEOs, and the other way around. As a result, neither a match between the best CEO and any of the 7% smallest firms nor a match between the largest firm and a CEO with ability below the 50th quantile of the ability distribution would be sustainable. The distribution of matches with negative surplus reflects asymmetric division of market power between firms and executives. With so few vacancies available, only 27% of the most outstanding CEOs will ever decline a job with the least productive firm. Yet more than twice as many (58%) of the firms will not be willing to work with CEOs at the bottom of ability distribution. Good CEOs are

generally happy to match with smaller firms because employment boosts their chances to receive a better offer, while smaller firms are weary of such matches knowing they won't be able to retain a stellar executive when a poacher approaches her.

Because of production complementarities, the surplus function is not necessarily monotonic in its arguments. Surplus is upward sloping in the areas that correspond to higher values of firm market value and CEO ability. This is where either party that is sufficiently well endowed with either market value or talent can always make better use of its own resources by bringing in more of the complementary input from another party. For example, the output produced by big firms would be constrained by mediocre management so much that they would only gain by hiring more talented managers even if it costs more in wages.

However the surplus function is not monotonic where one of the inputs is set at relatively low level, thus constraining the overall potential of possible matches. Consider a median executive who produces maximum surplus when matched with the 51th percentile firm. In the process of job-to-job transitions, this candidate could increase match surplus by switching to firms with higher market values up to the 51th firm percentile and decrease thereafter. Although the median CEO would make a feasible match with any firm on the market, in the process of job-to-job transitions they would move towards the surplus maximizing firms at the center of market value distribution. This process therefore may involve movements to better as well as to worse firms. It is also important to notice that the process of job mobility towards matches with better surplus does not necessarily lead to increases in the CEO compensation. In fact 25% of job-to-job transitions are associated with short term wage loss, as executives take into account future job prospects in addition to the current wages. We look further into the wage determination process in the next section.

5.3 Determinants of wage growth

We know from the data that the CEO compensation growth at a rate of approximately 4% per annum. This growth rate is closely matched by our model. In this section we discuss factors that impact the observed growth of executive wages.

In equilibrium, wage growth is generated through two main channels. The first channel is competition for CEO talent among firms that takes place through headhunting and poaching. Wages increase either as firms have to respond to counteroffers, or as executives move to better matches. If we assume that transitions between jobs are impossible by setting relative search intensity of incumbent CEOs to zero, the wage growth rates decrease by approximately one fourth, from 3.9 to 3.0%. Therefore, our estimates indicate that a quarter of wage growth in equilibrium is due to headhunting and poaching. Notice, however, that in the absence of CEO job mobility equilibrium average compensation will be 3 percent higher, and the starting offers will increase to 5% in order to compensate for limited promotion prospects. Probably the most striking difference is that the lowest compensation in the data would have gone up by 60%.

The second channel that is responsible for the remaining wage growth is shocks to the market value of the firms. These shocks may either increase or decrease wages. We are especially interested in the increases that are due to the CEOs extracting additional surplus when the firm experiences periods of growth, which we identify as CEO entrenchment in our model. We evaluate the impact of this change by setting the bargaining parameter of incumbent CEOs to $\beta_1 = 0$. Without surplus extraction, wage changes caused by market value shocks are dominated by negative changes that occur when firms find themselves unable to pay previously negotiated compensations, and have to offer executives full match surplus as the only way to keep them (albeit at lower wage). As a result, the overall wage growth turns negative but very close to zero. Similar to the case with headhunting, we find that the average wage would go up by 11 percent, the starting wage by 15% and the minimum accepted wage by a dramatic 80%. In a less extreme case we could assume that CEOs are able to extract surplus, but their bargaining power does

not change after appointment, that is, $\beta_1 = \beta_0$. In this latter case, the decrease in wage growth would be similar to that when we eliminated headhunting, approximately 25% of the original growth.

Based on these experiments, we conclude that the ability of CEOs to obtain a share in the additional match surplus is the most important determinant of the CEO wage growth. In terms of their impact on wage growth, complete elimination of headhunting is equivalent to a 14 percentage points decrease of the bargaining power of incumbent CEOs. The full annual wage growth rate of 3.9% is completely wiped out when there are no job-to-job transitions and the incumbents can only retain 23% of newly generated surplus, a half of their initial bargaining power. Setting the bargaining power of incumbent CEOs to $\beta_1 = 0.375$, which is roughly a half of our estimate, would equate the real growth of executive wages to that of the general workforce, which is around 1% per year.

This makes us think of the CEO entrenchment as the main explanation of the compensation growth. However, we notice from the data that poaching and benchmarking account for a substantial part of the executive compensation growth. Recent empirical evidence suggests a substantial increase of CEO mobility between firms within and across industries. As such moves become more acceptable, availability of the outside options increases the levels of compensation. Overall, we expect that the relative role of headhunting is likely to increase over time.

In addition, we explore several possible channels that could promote wage growth in transition between different equilibria, rather than focusing on the channels that work in the steady state. The candidate explanations that we explore are concentration of capital and increase in the size of the largest firms; complementarity between executive talent and firm value; CEO entrenchment reflected by the cost of recruitment; and matching efficiency.

First, we explore the relationship between firm size and executive compensation following Gabaix and Landier (2008) who find that the increase in market capitalization of the top US public companies largely explains the growth of CEO pay over the same

period. We run two counterfactuals setting up the parameters of firm size distribution so that the two resulting equilibrium distributions of market values matched the data in 1994 and 2017, respectively. Over this period market capitalization has increased by close to 500% and CEO compensation almost doubled. We find that observed growth of market capitalization alone accounts for approximately one third of executive pay growth. This is a large fraction, yet apparently there are other factors that contribute to the growth of executive compensation. Firm growth is endogenous and may be related to factors that simultaneously contribute to the growth of executive compensation but are outside our model. However, within the model we can still explore several directions that capture contemporary practices of executive compensation and are commonly thought to contribute to the CEO pay increase. To further this analysis, we explore a possibility that markets experienced a series of positive shocks to the firm market values without going through shifts in the underlying distribution of firm values. We use the model solution to simulate a dataset in which shocks to the firm market values are constrained to be only positive. While behavior of firms and executives takes into account the possibility of market values going down, this possibility never arrives in the simulations. We find that sustained growth in market values would increase CEO turnover and suppress wages.

The second factor that determines executive wages is complementarity between firm size and managerial talent. We saw earlier that production complementarities may result in substantial surplus gains, especially at the higher end of the distributions. We now have to see how these surplus gains are reflected in wage setting mechanisms. To explore this channel, we vary complementarity parameter ρ and record changes in the wage distribution under alternative scenarios. We find that higher values of complementarity increase wage dispersion, but decrease the average amount of compensation. For example, to generate a 5% increase in compensation that is roughly the level of annual compensation growth observed between years 2009 and 2018 we would need a huge increase in parameter ρ from -0.95 to -0.55. The main mechanism behind this change is that at higher levels of complementarity executives are willing to accept lower wages in

anticipation of high payoffs from renegotiation of their contracts: the starting wage would have to increase by 10%. Although these changes are consistent with the data in terms of direction of the changes, we don't think it is likely that changes in the production complementarities over the studied period could be high enough to justify the observed wage growth.

5.4 The costs of search frictions

In the absence of frictions complementarity between firm and management inputs would imply perfect assortative matching, as we know from assignment models of executive compensation. Search frictions limit the extent of positive assortative matching and cause unemployment. The degree of sorting depends on the magnitude of frictions. We quantify potential gains from removing the search frictions by solving a frictionless dynamic assignment mode and comparing welfare in the decentralized frictional and frictionless equilibria.

We define welfare as the sum of total output and production of non-CEO executives, net of the recruitment cost. Welfare in the estimated decentralized frictional equilibrium is normalized to 100. In this experiment we keep the number of firms and candidates in the economy fixed at their estimated levels as in decentralized equilibrium, and assign executives to the firms ignoring search frictions so that the resulting allocation $h^*(x, y)$ maximizes the welfare function,

$$h^*(x, y) = \arg\max_{h(x,y)} \int f(x, y)h(x, y)dxdy + \int b(x)u(x)dx - \int c(y)v(y)dy.$$

This counterfactual imposes perfect assortative matching, identifies the mismatch effect of frictions and gives an upper bound on the potential benefits from eliminating the frictions in a hypothetical world with centralized assignment of the CEOs. In the new distribution of matches firms and executives are perfectly positively sorted.

The results of the experiment are summarized in column 2 of Table 4 that contains a

summary of all welfare experiments. The first experiment shows that there are substantial losses caused by search frictions: improvement of assignment technology has a potential to increase welfare by up to 67%. The change is mainly driven by improvements in match quality made possible by improved sorting that almost triple the overall output. In addition, frictionless welfare benefits from lack of recruitment cost and full occupancy of the available posts. These positive effects are somewhat offset by decrease in the production of lower level executive who are on average less capable in the frictionless equilibrium that skimmed the best talent to hold CEO jobs.

The average CEO compensation is 1.64 times lower in the frictionless environment. This difference results from the elimination of wage growth channels. With search frictions the best CEOs will often accept jobs at smaller firms and use their position to improve wages by generating counteroffers and withholding a share of firm growth. By the time one of the top firms approaches them, they would have climbed high enough to solicit an offer in excess of their starting share β. In the world without frictions they do not have this leverage and have to satisfy themselves with the initial offer, knowing that it already comes from the best firm they can form a match with. The same happens throughout the entire ability ladder.

The benchmark frictionless equilibrium result described here is quite extreme and not at all practical in terms of policy recommendations. It suggests, nevertheless, that efficiency losses due to search frictions are potentially considerable. Even small improvements in search technology and job allocation, therefore, are likely to yield sizable welfare gains. While such improvements are evidently of practical interest, it is beyond the scope of this paper to discuss their costs and feasibility.

6 Conclusions

As evidenced by the sharp increase in executive turnover in 2019, executives have to search for jobs, and firms have to search for executives, sometimes on short notice. Such

searching and matching can occur even if the CEO delivered large profits for the company, as in the case of MacDonalds, where the CEO was fired for ethical violations. It stands to reason, therefore, that a potentially valuable perspective on the CEO market can be gleaned from a model that can evaluate dynamic search-match frictions, in a non-perfectly competitive market.

One particular concern that boards of directors have when setting compensation packages, is that a competitor may pay more to poach their CEO, or a CEO may leave to work in a private equity firm. This external pressure encourages boards to boost compensation packages, thereby ensuring that their CEOs are not underpaid relative to the market. Extant research has not developed a framework to quantify the role of labour market frictions and peer pressure in the determination of executive pay, and our paper begins to address this gap.

We develop and estimate a structural equilibrium model of executive hiring and compensation, which incorporates a nontrivial role for market frictions. We utilise a framework of search and matching, and contribute to that literature by providing a setting where identification of firm productivity is reasonable, due to the data we gather on firm characteristics, and the CEO-firm relationship which is one-to-one. We evaluate the extent to which market frictions impinge on the efficiency and level of executive hiring. We document several interesting results. First, we find that complementarity between firm size and executive talent, which explains much of the growth in executive compensation in a frictionless environment, only accounts for about one third of observed wage dispersion. The remaining two thirds of observed wage dispersion is attributed to the market frictions. Second, the welfare effects of market frictions are considerable. When frictions are introduced, the match quality between CEOs and firms drops by nearly two thirds. Moreover, relative to the benchmark frictionless case, welfare in the model with frictions is 40% lower. The finding that CEO compensation is to a considerable degree affected by frictions, suggests that efficient monitoring of compensation depends on several factors, including the firm, the specific CEO, the market structure, and the precise nature of the

frictions. In order to balance these diverse and potentially countervailing factors, this setting is likely to require sophisticated governance strategies.

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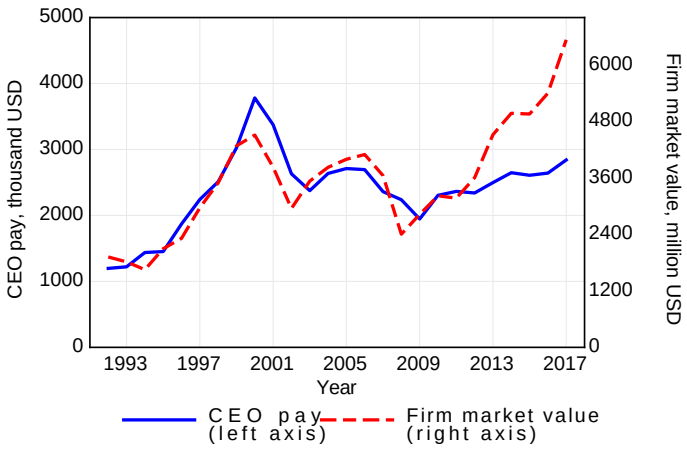
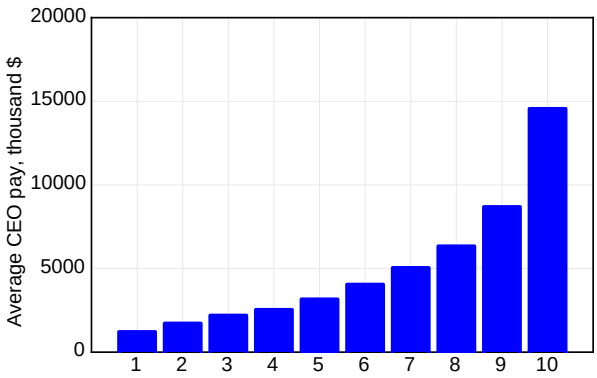
Figures and Tables

Figure 1: Executive compensation and firm market values



Notes: Based on ExecuComp series for executive compensation (tdc1) and market value (mktval), 1993-2016. All amounts are deflated using CPI. Sample size $n = 41,183$.

Figure 2: Executive compensation by firm market value



Notes: Average executive compensation by deciles of firm market value. Based on ExecuComp series for executive compensation (tdc1) and market value (mktval), 1993-2016. All amounts are expressed in real 2018 dollars using CPI deflator. Sample size $n = 41,183$.

Figure 3: Model fit, employment and labor force transitions

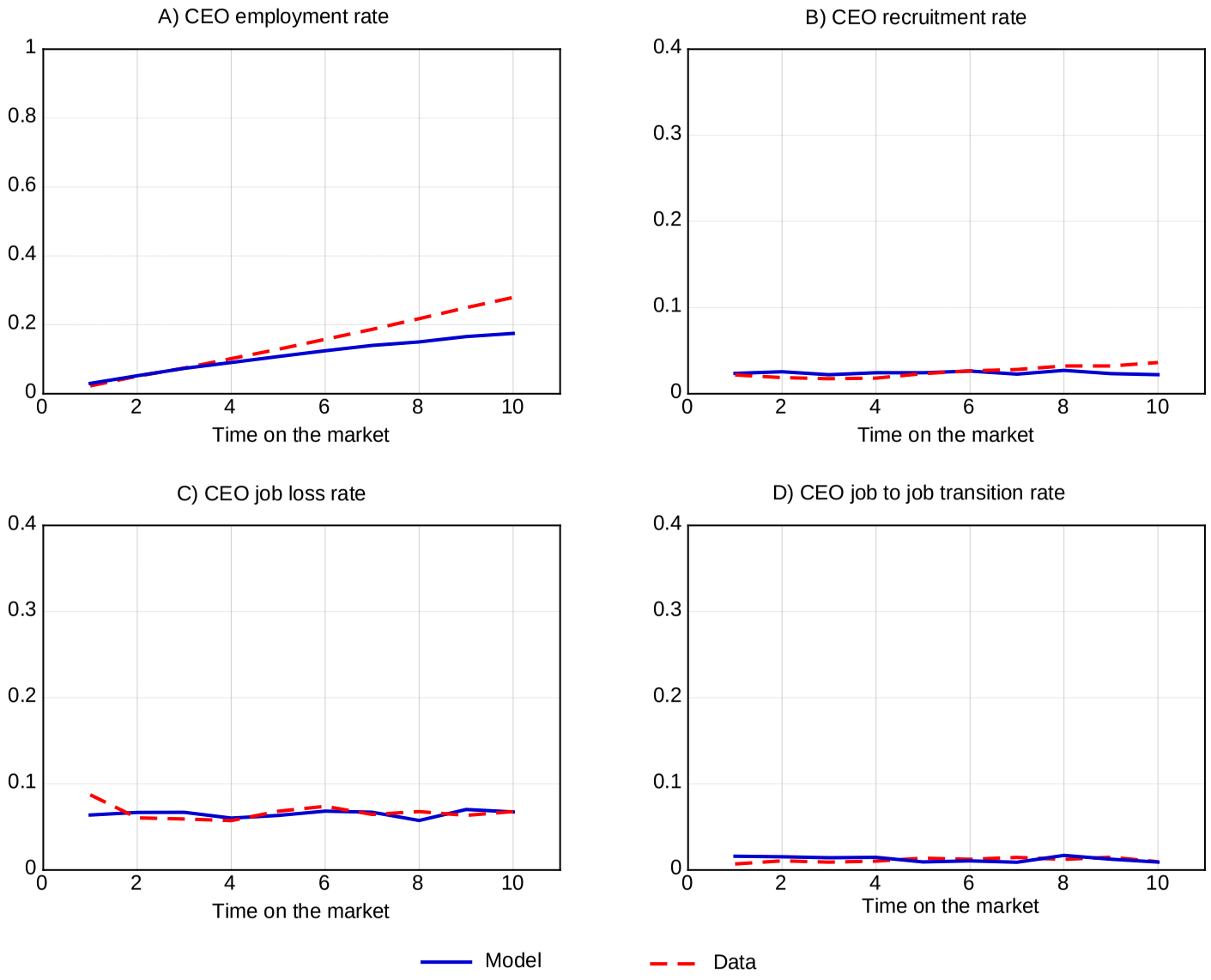


Figure 4: Model fit, wages

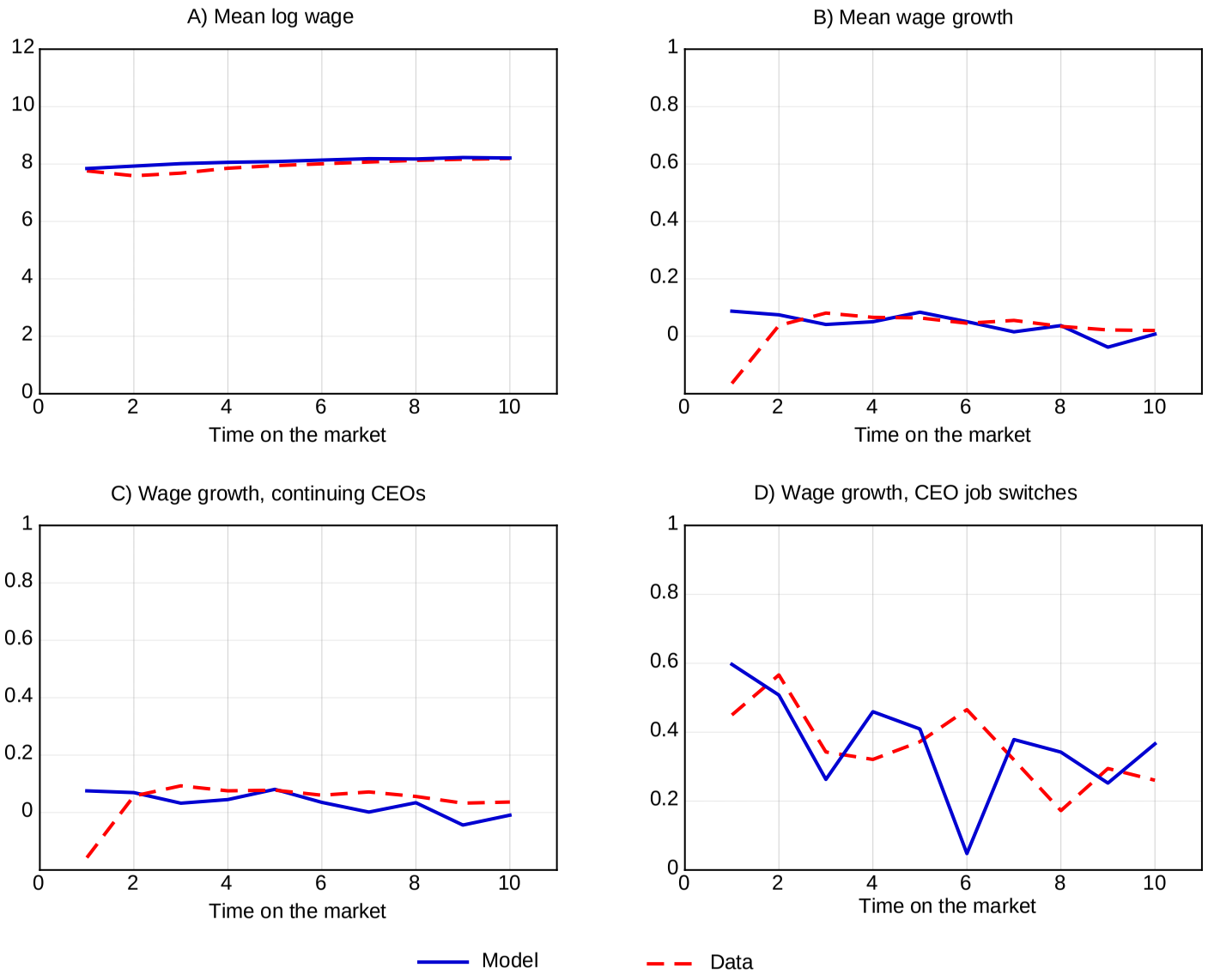


Figure 5: Equilibrium distribution of matches

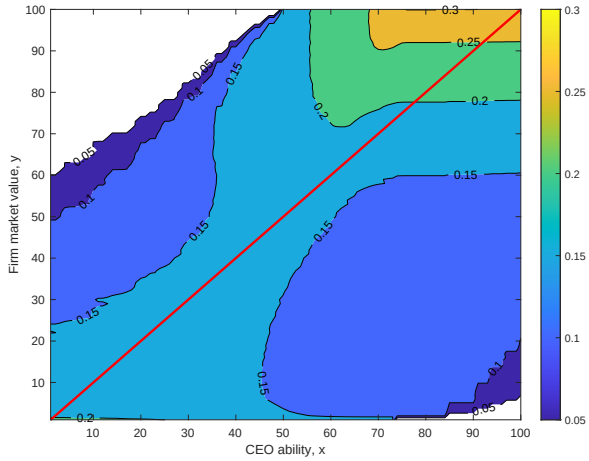


Figure 6: Surplus function

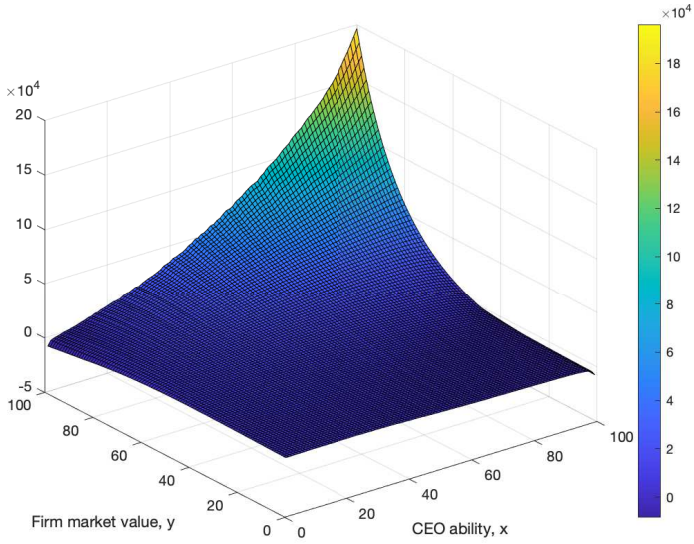


Table 1: Descriptive statistics

Variable	Firm market value range:		
	Quartile 1	Quartiles 2-3	Quartile 4
Annual CEO compensation, thousand USD			
Mean	2,043	4,735	13,093
Median	1,448	3,396	9,366
Standard deviation	2,927	6,313	21,002
Firm market value, million USD			
Mean	365	2,303	32,167
Median	367	1,872	14,602
Standard deviation	190	1,389	55,321
Annual compensation of non-CEO executives, thousand USD			
Mean	855	1,706	4,760
Median	629	1,223	3,084
Standard deviation	978	2,282	7,905
Annual CEO compensation growth, %	-0.9 (65.7)	6.9 (63.3)	6.2 (60.6)
Wage change from becoming a CEO, %	7.4 (84.5)	25.7 (81.0)	33.4 (72.5)
Wage change from switching CEO jobs, %	-44.2 (168)	57.6 (242)	42.7 (207)
Annual probability of CEO job loss, %	16.8 (37.4)	12.8 (33.4)	12.9 (33.5)
Annual probability of CEO job switch, %	0.65 (8.0)	1.0 (10.0)	1.1 (10.4)
Annual probability of CEO job finding, %	0.24 (4.8)	0.34 (5.8)	0.15 (3.9)
Duration of the CEO appointments, years			
Mean	7.6	7.5	6.5
Median	5.0	6.0	5.0
Standard deviation	7.8	6.8	5.5
Number of firms	2,130	2,993	1,184
Number of executives	19,865	31,641	14,665
Number of CEOs	3,365	5,125	2,317
Number of CEO-firm matches	2,617	3,598	1,567
Number of completed spells	1,758	2,454	1,086

Notes: Merged ExecuComp and Compustat database, 1992-2018. All monetary values in real 2018 dollars. Standard deviations are shown in the parentheses.

Table 2: Estimates of the structural parameters

Parameter	Estimate	Standard error
Matching efficiency, η	0.041	0.015
New CEO bargaining power, β_0	0.503	0.141
Incumbent CEO bargaining power, β_1	0.692	0.051
Relative search intensity among employed CEOs, s	1.359	0.230
Probability of exogenous job destruction, ξ	0.007	0.004
Probability of a productivity shock, δ	0.147	0.013
Vacancy cost, c	6.014	2.085
The value of CEO outside option, b	0.269	0.039
CEO talent distribution:		
Mean, μ_x	6.536	0.732
Standard deviation, σ_x	3.226	0.332
Minimum, x_{min}	139.648	11.718
Maximum, x_{max}	14000	1130
Market value distribution:		
Mean, μ_y	7.820	0.317
Standard deviation, σ_y	1.548	0.029
Minimum, y_{min}	58.297	0.703
Maximum, y_{max}	68400	3900
Parameters of the production function:		
Technology, A	-1.164	0.091
Weight on the CEO talent, α	0.689	0.051
Elasticity of substitution, ρ	-0.806	0.237
Mortality rate, μ	0.009	0.000
χ^2 statistic = 485, degrees of freedom = 76		

Notes: MCMC estimates of the structural parameters. Estimation uses diagonal weighting matrix.

Table 3: Model fit

Moment	Model	Data	S.D.
Employment rate	0.107	0.102	0.302
Job finding probability	0.028	0.022	0.147
Job loss probability	0.074	0.066	0.248
Job to job transition probability	0.015	0.012	0.111
Wage, mean	8.01	7.99	1.03
Starting wage, mean	7.875	7.872	1.22
Wage, minimum	4.108	4.323	0.0004
Wage, maximum	11.089	11.113	0.0007
Average wage growth	0.065	0.065	0.578
Average wage growth, EE	0.060	0.065	0.664
Average wage growth, JJ	0.275	0.240	1.071
Average career length	6.60	6.59	5.58
Firm market value, mean	7.56	7.39	1.40
Firm market value, s.d.	1.40	1.40	0.00
Firm market value, min	4.07	4.07	0.0004
Firm market value, max	10.99	11.07	0.0016
Within-firm variance of market values	0.161	0.169	0.205
Wage/size regression	0.303	0.285	0.013
Vacancies to jobs	0.048	0.050	0.048

Means taken over the first ten years in the data.

Table 4: Welfare analysis

	Model	Frictionless equilibrium
Total output	43.92	123.21
Non-CEO production	66.18	43.36
Recruitment cost	10.10	0.00
Welfare	100.00	166.57
CEO employment rate	12.27	12.91
Vacancies to unemployment	0.73	0.00
Average compensation, CEOs	8.69	7.06
Average compensation, other executives	5.34	4.44
Match quality (output per match)	0.036	0.095

Welfare in the decentralized frictional equilibrium estimated by the model (column 1) is normalized to 100. Column (2) is frictionless equilibrium with the number of firms and executives fixed at the level of decentralized estimated equilibrium.

A Technical appendix

A.1 Data: variables and sample construction

We utilize data from two subfiles within ExecuComp, namely, anncomp (CEO compensation details and CEO identifier) and codirfin (director information).

Anncomp contains several variables, of which we extract the following, where dataset names are in brackets: CEO age (age); dates that determine the beginning (becameceo) and end (leftofc) of the CEO tenure; bonus (bonus); CEO flag (ceoann); company name (coname); firm ID (gvkey); salary (salary); number of shares owned by the CEO, excepting options (shrown_excl_opts); percentage of shares owned by the CEO (shrown_excl_opts_pct);[2] total compensation, with options and stock awards valued at grant date (tdc1); current compensation (total_curr); and fiscal year (year). From the subfile codirfin we obtain observations on firm ID (gvkey); return on assets (roa); net sales (sales); shares outstanding (shrsout); and fiscal year (year).
Further company level data are extracted from the Compustat database, using the Compustat annual database with monthly updates, which is denoted 'funda' in the file wrds/comp/sasdata/m_na . We extract the following variables: total asset value (at); book value (ceq); common shares outstanding (csho); current debt (dlc); long term debt (dltt); dividend payments (dvc); fiscal year (fyear); firm identifier (gvkey); inventories (invt); gross plant and physical equipment (ppegt); and closing price for the fiscal year-end (prcc_f). These variables are combined with the ExecuComp data above on the basis of gvkey and year, in order to obtain our working dataset.

A.2 Details for value functions

A.2.1 CEO candidates without jobs

The present value of unemployment is a sum of the value of outside option and the expected gain from finding a job with firm of type y'. The added value of taking a job with firm of type y' is given by $w_0(x, y') + W_1(x, y', w) - W_0(x) = w_0(x, y') + \beta_0 S(x, y')$. Potential match will only be consummated if firm and executive can generate non-negative surplus, hence the value of unemployment is given by

$$(r + \mu)W_0(x) = b(x) + \kappa \int\limits_{y' \in \mathcal{A}(x)} \big[w_0(x, y') + \beta_0 S(x, y')\big] v(y') dy', \tag{A.2.1}$$

[2] ExecuComp does not report percentage of shares when this quantity is less than 1%. In most of these cases the number of shares held are available, so we divide the number of shares by the total shares outstanding in order to obtain percentage shareholding.

where $\mathcal{A}(x) = \{y : S(x, y) \geq 0\}$ is a set of firms that can form a sustainable match with a candidate of type x. This is equation (4) in the text.

A.2.2 Firms with vacant CEO positions

The four components of the value function are obtained as follows.

1. The ongoing cost of maintaining the vacancy open, $-c$.

2. The change of state value due to the arrival of productivity shocks that shifts firm value from y to y'. The shocks come from distribution with density $\gamma(y)$ at rate δ:

$$\delta \int_{y'} \left(\Pi_0(y') - \Pi_0(y)\right) \gamma(y') dy'.$$

3. The gains from filling a vacant post with unemployed candidate of type x. The probability of firm meeting a candidate is given by $\kappa u(x)$. $\Pi_1(w, x, y) - \Pi_0(y) - w_0(x, y) = (1 - \beta_0)S(x, y) - w_0(x, y)$ is the change in state value, corresponding to the firm's share of surplus. We integrate over all matches that generate non-negative surplus,

$$\begin{aligned} &\kappa \int\limits_{x \in \mathcal{B}(y)} \big[\Pi_1(w, x, y) - \Pi_0(y) - w_0(x, y)\big] u(x) dx \\ &= \kappa \int\limits_{x \in \mathcal{B}(y)} \big[(1 - \beta_0)S(x, y) - w_0(x, y)\big] u(x) dx, \end{aligned}$$

   where $\mathcal{B}(y) = \{x : S(x, y) \geq 0\}$ is a set of executives that can form a sustainable match with a firm of type y.

4. The gains from filling a vacant post with headhunted candidate of type x. The probability of firm meeting such candidate is given by $s\kappa h(x, y')$, where $h(x, y')$ gives the distribution of the current CEOs of type x employed by firms of type y'. The match will only be consummated if it can create surplus higher than the surplus of the current match of the headhunted CEO. The set of sustainable matches is defined as $\mathcal{C}(y) = \{(x, y') : S(x, y) \geq S(x, y')\}$. The firm's share of surplus in this case is $[S(x, y) - S(x, y') - w_0(x, y)]$. Hence, this component is given by

$$s\kappa \iint\limits_{(x,y') \in \mathcal{C}(y)} \big[S(x, y) - S(x, y') - w_0(x, y)\big] h(x, y') dx dy'.$$

Taken together, this gives the value function for firms with vacant CEO position (equation (5) in the text),

$$
\begin{aligned}
r\Pi_0(y) = -c + \delta \int_{y'} [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' \\
+\kappa \int_{x' \in \mathcal{B}(y)} \big[(1-\beta_0)S(x',y) - w_0(x',y)\big]u(x')dx' \\
+s\kappa \iint_{(x',y') \in \mathcal{C}(y)} \big[S(x',y) - S(x',y') - w_0(x',y)\big]h(x',y')dx'dy'.
\end{aligned}
$$

A.2.3 Employed CEO's

The state value for the CEO of type x heading a firm with market value y at wage w is $W_1(w,x,y)$. There are five components in the value function of employed CEOs that can be rearranged to obtain equation (7).

1. The value of staying in the current state at the current wage, w.

2. The loss of value due to death, $-\mu W_1(w,x,y)$.

3. The loss of value if job is destroyed. Exogenous job destruction shocks arrive at rate ξ and move the CEO to the unemployment state,

$$\xi[W_0(x) - W_1(w,x,y)].$$

4. Change of state value due to the arrival of a productivity shock that changes firm value from y to y'. This includes the following possibilities. In the integration limits we use notation $\Delta W' = W_1(w,x,y') - W_0(x)$ and $S' = S(x,y')$.

   (a) Match is no longer viable and is endogenously destroyed, $S(x,y') < 0$. The CEO returns to the pool of candidates:

$$
\begin{aligned}
&\int_{y':S'<0} \big[W_0(x) - W_1(w,x,y)\big]\gamma(y')dy' \\
&= \big[W_0(x) - W_1(w,x,y)\big] \int_{y':S'<0} \gamma(y')dy'.
\end{aligned}
$$

   (b) The new surplus is non-negative, $S(x,y') \geq 0$, but the CEO's wage exceeds the total surplus, $W_1(w,x,y') - W_0(x) > S(x,y')$. The change of state value reflects

renegotiation of the new contract that satisfies $W_1(w', x, y') = W_0(x) + S(x, y')$:

$$\int\limits_{y':0\leq S'<\Delta W'} [S(x, y') + W_0(x) - W_1(w, x, y)]\gamma(y')dy'$$
$$= [W_0(x) - W_1(w, x, y)] \int\limits_{y':0\leq S'<\Delta W'} \gamma(y')dy' + \int\limits_{y':0\leq S'<\Delta W'} S(x, y')\gamma(y')dy'.$$

(c) The new surplus is non-negative but lower than the original surplus, $0 \leq S(x, y') < S(x, y)$ and the CEO's wage contract falls below the outside option value, $W_1(w, x, y') - W_0(x) < 0$. The value reflects renegotiation with the new contract that satisfies $W_1(w', x, y') = W_0(x)$:

$$[W_0(x) - W_1(w, x, y)] \int\limits_{y':0\leq S'<S,\Delta W'<0} \gamma(y')dy'.$$

(d) The new surplus is higher than the original surplus, $S(x, y') \geq S(x, y)$. The wage contract is renegotiated to guarantee that the CEO receives at least a fraction β_1 of the new surplus, $W_1(w', x, y') = \max\{W_1(w, x, y'), W_0(x) + \beta_1 S(x, y')\}$:

$$\int\limits_{y':S'\geq S,\Delta W'\leq S'} [\max\{W_1(w, x, y'), W_0(x) + \beta_1 S(x, y')\} - W_1(w, x, y)]\gamma(y')dy'.$$

(e) The surplus is non-negative but did not increase, $0 \leq S(x, y') \leq S(x, y)$, the adjusted value of the job for the CEO is above the value of the outside option, and firm can afford paying previously agreed compensation. There is no change to the wage contract.

$$\int\limits_{y':0\leq S'<S,0\leq\Delta W'\leq S'} [W_1(w, x, y') - W_1(w, x, y)]\gamma(y')dy'$$

Collect the terms taking into account the arrival rate δ:

$$\delta\Bigg[\Big[W_0(x) - W_1(w,x,y)\Big]\Bigg(\int\limits_{y':S'<0}\gamma(y')dy' + \int\limits_{y':0\le S'<\Delta W'}\gamma(y')dy' + \int\limits_{y':0\le S'<S,\Delta W'<0}\gamma(y')dy'\Bigg)$$
$$+\int\limits_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy'$$
$$+\int\limits_{y':S\le S',\Delta W'\le S'}[\max\{W_1(w,x,y'),W_0(x)+\beta_1 S(x,y')\} - W_1(w,x,y)]\gamma(y')dy'$$
$$+\int\limits_{y':0\le S'<S,0\le\Delta W'\le S'}[W_1(w,x,y') - W_1(w,x,y)]\gamma(y')dy'\Bigg],$$

then add and subtract the term

$$W_0(x)\Bigg(\int\limits_{y':S\le S',\Delta W'\le S'}\gamma(y')dy' + \int\limits_{y':0\le S'<S,0\le\Delta W'\le S'}\gamma(y')dy'\Bigg)$$

to complete the integral:

$$\delta\Bigg[W_0(x) - W_1(w,x,y) + \int\limits_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy'$$
$$+\int\limits_{y':S\le S',\Delta W'\le S'}\max\{W_1(w,x,y') - W_0(x),\beta_1 S(x,y')\}\gamma(y')dy'$$
$$+\int\limits_{y':0\le S'<S,0\le\Delta W'\le S'}[W_1(w,x,y') - W_0(x)]\gamma(y')dy'\Bigg]$$
$$=\delta\Bigg[W_0(x) - W_1(w,x,y) + \int\limits_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy'$$
$$+\int\limits_{y':S\le S',\Delta W'<\beta_1 S'}\beta_1 S(x,y')\gamma(y')dy'$$
$$+\int\limits_{y':\beta_1 S'\mathbb{1}\{S\le S'\}\le\Delta W'\le S'}[W_1(w,x,y') - W_0(x)]\gamma(y')dy'\Bigg]$$

5. The final part is change of state value due to headhunting. Employed CEOs meet competing employers with market value y' at rate $skv(y')$. The outcomes of headhunting include two cases when the contract is renegotiated.

(a) The new match can generate higher surplus, $S(x, y') > S(x, y)$. The CEO moves and receives the new wage w' that solves $W_1(w', x, y') - W_0(x) = S(x, y)$. The corresponding change of the state value is given by

$$\int\limits_{y':S\leq S'} \big[S(x, y) + W_0(x) - W_1(w, x, y) + w_0(x, y')\big]v(y')dy'.$$

(b) The new surplus is lower than the current surplus, but higher than the current contract value, $W_1(w, x, y) - W_0(x) < S(x, y') < S(x, y)$. The CEO stays with the current firm and renegotiates the compensation to $W_1(w', x, y) - W_0(x) = S(x, y')$. The change of state value is

$$\int\limits_{y':\Delta W\leq S'<S} \big[S(x, y') + W_0(x) - W_1(w, x, y)\big]v(y')dy'.$$

Collect the terms and taking into account the arrival rate $s\kappa v(y)$:

$$s\kappa\Bigg[\int\limits_{y':S\leq S'} \big[S(x, y) + W_0(x) - W_1(w, x, y) + w_0(x, y')\big]v(y')dy'$$
$$+ \int\limits_{y':\Delta W\leq S'<S} \big[S(x, y') + W_0(x) - W_1(w, x, y)\big]v(y')dy'\Bigg]$$
$$= s\kappa\Bigg[[W_0(x) - W_1(w, x, y)]\int_{y'\in\mathcal{D}(w,x,y)} v(y')dy'$$
$$+ \int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x, y), S(x, y')\} + w_0(x, y')\mathbb{1}\{S(x, y) \leq S(x, y')\}\big]v(y')dy'\Bigg],$$

where $\mathcal{D}(w, x, y) = \{y' : W_1(w, x, y) - W_0(x) < S(x, y')\}$.

Now collect the four terms to get the value of employment state for a CEO.

$$
\begin{aligned}
(r+\mu)W_1(w,x,y) &= w + \xi\big[W_0(x) - W_1(w,x,y)\big] \\
&\delta\Bigg[W_0(x) - W_1(w,x,y) + \int\limits_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy' \\
&\qquad + \int\limits_{y':S\le S',\Delta W'<\beta_1 S'} \beta_1 S(x,y')\gamma(y')dy' \\
&\quad + \int\limits_{y':\beta_1 S'\mathbb{1}\{S\le S'\}\le \Delta W'\le S'} [W_1(w,x,y') - W_0(x)]\gamma(y')dy'\Bigg] \\
&\quad + s\kappa\Bigg[[W_0(x) - W_1(w,x,y)]\int_{y'\in\mathcal{D}(w,x,y)} v(y')dy' \\
&+ \int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x,y),S(x,y')\} + w_0(x,y')\mathbb{1}\{S(x,y)\le S(x,y')\}\big]v(y')dy'\Bigg] \\
&= w + \left(\xi + \delta + s\kappa\int_{y'\in\mathcal{D}(w,x,y)} v(y')dy'\right)[W_0(x) - W_1(w,x,y)] \\
&\delta\Bigg[\int\limits_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy' + \int\limits_{y':S\le S',\Delta W'<\beta_1 S'} \beta_1 S(x,y')\gamma(y')dy' \\
&\qquad + \int\limits_{y':\beta_1 S'\mathbb{1}\{S\le S'\}\le \Delta W'\le S'} [W_1(w,x,y') - W_0(x)]\gamma(y')dy'\Bigg] \\
&+ s\kappa\int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x,y),S(x,y')\} + w_0(x,y')\mathbb{1}\{S(x,y)\le S(x,y')\}\big]v(y')dy'.
\end{aligned}
$$

Use the value of unemployment state to rearrange the terms:

$$
\begin{aligned}
(r+\mu)\big[W_1(w,x,y)-W_0(x)\big] &= w-(r+\mu)W_0(x) \\
+\left(\xi+\delta+s\kappa\int_{y'\in\mathcal{D}(w,x,y)} v(y')dy'\right)&\left[W_0(x)-W_1(w,x,y)\right] \\
\delta\Bigg[\int_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy' &+ \int_{y':S\le S',\Delta W'<\beta_1 S'} \beta_1 S(x,y')\gamma(y')dy' \\
+\int_{y':\beta_1 S'\mathbb{1}\{S\le S'\}\le\Delta W'\le S'} &[W_1(w,x,y')-W_0(x)]\gamma(y')dy'\Bigg] \\
+s\kappa\int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x,y),S(x,y')\}&+w_0(x,y')\mathbb{1}\left\{S(x,y)\le S(x,y')\right\}\big]v(y')dy'.
\end{aligned}
$$

Regroup further to get

$$
\begin{aligned}
\left[r+\mu+\delta+\xi+s\kappa\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy'\right]&\left[W_1(w,x,y)-W_0(x)\right] \\
&= w-b(x)-\kappa\beta_0\int_{y'} S(x,y')^+v(y')dy' \\
+\delta\Bigg[\int_{y':0\le S'<\Delta W'} S(x,y')\gamma(y')dy' &+ \int_{y':S\le S',\Delta W'<\beta_1 S'} \beta_1 S(x,y')\gamma(y')dy' \\
+\int_{y':\beta_1 S'\mathbb{1}\{S\le S'\}\le\Delta W'\le S'} &[W_1(w,x,y')-W_0(x)]\gamma(y')dy'\Bigg] \\
+s\kappa\int_{y'\in\mathcal{D}(w,x,y)} \big[\min\{S(x,y),S(x,y')\}&+w_0(x,y')\mathbb{1}\left\{S(x,y)\le S(x,y')\right\}\big]v(y')dy'.
\end{aligned}
$$

This is equation (6) in the text.

A.2.4 Firms with CEOs

The state value for the firm of type y managed ed by the CEO of type x at wage w is $\Pi_1(w,x,y)$. There are 4 components symmetric to those in the value function of employed CEOs.

1. The cost of paying the current wage, $-w$.

2. The loss of value if the CEO position is destroyed. Exogenous job destruction shocks arrive at rate ξ and turn occupied positions into vacancies,

$$\xi[\Pi_0(y)-\Pi_1(w,x,y)] = -\xi(1-\beta_0)S(x,y).$$

3. Change of state value due to the arrival of a productivity shock that changes firm value from y to y'. This includes four possibilities.

   (a) Match is no longer viable and is endogenously destroyed, $S(x, y') < 0$. In the integration limits, this condition is referred to below as $S < 0$. The firm reopens a vacancy:

   $$\int_{y':S<0} [\Pi_0(y') - \Pi_1(w, x, y)]\gamma(y')dy' = \int_{y':S<0} [\Pi_0(y') - \Pi_0(y) - (1-\beta)S(x, y)]\gamma(y')dy'.$$

   (b) The new surplus is non-negative, $S(x, y') \geq 0$, but the CEO's wage falls below the outside option value, $W_1(w, x, y') - W_0(x) < 0$. In the integration limits, this condition is referred to below as $\Delta W < 0$. The new state value reflects renegotiation that result in firm collecting the entire surplus, $S(x, y') = \Pi_1(w', x, y') - \Pi_0(y')$:

   $$\int_{y':S\geq 0,\Delta W<0} [S(x, y') + \Pi_0(y') - \Pi_0(y) - (1 - \beta_0)S(x, y)]\gamma(y')dy'.$$

   (c) The new surplus is non-negative, $S(x, y') \geq 0$, but the CEO's wage exceeds the total surplus, $W_1(w, x, y') - W_0(x) > S(x, y')$. After renegotiations, all surplus is collected by the CEO, $\Pi_1(w', x, y') - \Pi_0(y') = 0$:

   $$\int_{y':S\geq 0,\Delta W>S} [\Pi_0(y') - \Pi_0(y) - (1 - \beta_0)S(x, y)]\gamma(y')dy'.$$

   (d) The surplus is non-negative, $S(x, y') \geq 0$, the adjusted value of the job for the CEO is above the value of the outside option, and firm can afford paying previously agreed compensation,

   $$\int_{y':S\geq 0,\Delta W\leq S} [\Pi_0(y') - \Pi_0(y) + (1 - \beta_0)(S(x, y') - S(x, y))]\gamma(y')dy'$$

Collect the four terms taking into account the arrival rate δ:

$$\delta\Bigg[\int_{y':S<0}[\Pi_0(y')-\Pi_0(y)-(1-\beta_0)S(x,y)]\gamma(y')dy'$$
$$+\int_{y':S\geq 0,\Delta W<0}[S(x,y')+\Pi_0(y')-\Pi_0(y)-(1-\beta_0)S(x,y)]\gamma(y')dy'$$
$$+\int_{y':S\geq 0,\Delta W>S}[\Pi_0(y')-\Pi_0(y)-(1-\beta_0)S(x,y)]\gamma(y')dy'$$
$$+\int_{y':S\geq 0,\Delta W\leq S}[\Pi_0(y')-\Pi_0(y)+(1-\beta_0)(S(x,y')-S(x,y))]\gamma(y')dy'\Bigg],$$

and rearrange the terms:

$$\delta\Bigg[\int[\Pi_0(y')-\Pi_0(y)]\gamma(y')dy'-(1-\beta_0)S(x,y)+\int_{y':S\geq 0,\Delta W<0}S(x,y')\gamma(y')dy'$$
$$+\int_{y':S\geq 0,\Delta W\leq S}(1-\beta_0)S(x,y')\gamma(y')dy'\Bigg]$$
$$=\delta\Bigg[\int[\Pi_0(y')-\Pi_0(y)]\gamma(y')dy'-(1-\beta_0)S(x,y)$$
$$+\int_{y':S\geq 0,\Delta W\leq S}[S(x,y')-[W_1(w,x,y')-W_0(x)]^+\beta_0 S(x,y')]\gamma(y')dy'\Bigg].$$

4. The final part is change of state value due to headhunting. The CEO of a firm with market value y is headhunted at rate $skv(y)$. The outcomes of headhunting include two cases that affect the firm state value.

   (a) The new match can generate higher surplus, $S(x,y')>S(x,y)$. The CEO moves and the firm reopens a vacancy. The corresponding change of the state value is given by

   $$\int_{y':S(x,y')>S(x,y)}-(1-\beta_0)S(x,y)v(y')dy'.$$

   (b) The new surplus is lower than the current surplus, but higher than the current contract value, $W_1(w,x,y)-W_0(x)<S(x,y')<S(x,y)$. The CEO stays with the current firm and renegotiates the compensation so that more surplus goes

to the CEO, $W_1(w', x, y) - W_0(x) = S(x, y')$. The change of state value is

$$\int_{y':W_1(w,x,y)-W_0(x)<S(x,y')<S(x,y)} \big[\Pi_1(w', x, y) - \Pi_1(w, x, y)\big] v(y')dy'$$
$$= \int_{y':W_1(w,x,y)-W_0(x)<S(x,y')<S(x,y)} (1-\beta)[S(x, y') - S(x, y)]v(y')dy'.$$

Collect the terms and take into account the arrival rate $s\kappa v(y)$:

$$s\kappa\Bigg[\int_{y':S(x,y')>S(x,y)} -(1-\beta_0)S(x, y)v(y')dy'$$
$$+\int_{y':W_1(w,x,y)-W_0(x)<S(x,y')<S(x,y)} (1-\beta_0)[S(x, y') - S(x, y)]v(y')dy'\Bigg]$$
$$= -(1-\beta_0)s\kappa\Bigg[S(x, y)\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy'$$
$$-\int_{y'\in\mathcal{A}(w,x,y)} \mathbb{1}\left\{S(x, y) > S(x, y')\right\} S(x, y')v(y')dy'\Bigg],$$

where $\mathcal{A}(w, x, y) = \{y' : W_1(w, x, y) - W_0(x) < S(x, y')\}$.

Now collect the four terms to get the value of employment state for a firm with CEO.

$$
\begin{aligned}
r\Pi_1(w,x,y) = r\Pi_0(y) + (1-\beta_0)rS(x,y) = -w - \xi(1-\beta_0)S(x,y) \\
+\delta\Bigg[\int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' - (1-\beta_0)S(x,y) \\
+\int_{y':S\geq 0,\Delta W\leq S} [S(x,y') - [W_1(w,x,y') - W_0(x)]^+\beta_0 S(x,y')]\gamma(y')dy'\Bigg] \\
-(1-\beta_0)s\kappa\Bigg[S(x,y)\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy' \\
-\int_{y'\in\mathcal{A}(w,x,y)} \mathbb{1}\left\{S(x,y) > S(x,y')\right\} S(x,y')v(y')dy'\Bigg] \\
= -w - (1-\beta_0)\left(\xi + \delta + s\kappa\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy'\right)S(x,y) \\
+\delta\Bigg[\int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' \\
+\int_{y':S\geq 0,\Delta W\leq S} [S(x,y') - [W_1(w,x,y') - W_0(x)]^+\beta_0 S(x,y')]\gamma(y')dy'\Bigg] \\
-s\kappa\int_{y'\in\mathcal{A}(w,x,y)} \mathbb{1}\left\{S(x,y) > S(x,y')\right\} S(x,y')v(y')dy'.
\end{aligned}
$$

Rearrange the terms:

$$
\begin{aligned}
\left[r + \delta + \xi + s\kappa\int_{y'\in\mathcal{A}(w,x,y)} v(y')dy'\right](1-\beta_0)S(x,y) = \\
-w + \delta\Bigg[\int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' \\
+\int_{y':S\geq 0,\Delta W\leq S} [S(x,y') - [W_1(w,x,y') - W_0(x)]^+\beta_0 S(x,y')]\gamma(y')dy'\Bigg] \\
-s\kappa\int_{y'\in\mathcal{A}(w,x,y)} \mathbb{1}\left\{S(x,y) > S(x,y')\right\} S(x,y')v(y')dy'.
\end{aligned}
$$

A.2.5 The match surplus equation

Based on the value functions above, we can further derive a Bellman equation for the match surplus that does not depend on the current wages (equation (7) in the text). First, consider the present value of the match output $P(x, y)$ as the sum of flow output and expected changes due to job destruction, head hunting and productivity shocks. Next, use the surplus definition $S(x, y) = P(x, y) - \Pi_0(y) - W_0(x)$ to substitute out joint production and the value of unmatched states.

$$
\begin{aligned}
rP(x,y) &= f(x,y) + \xi[\Pi_0(y) + W_0(x) - P(x,y)] + \mu[\Pi_0(y) - P(x,y)] \\
&+ s\kappa \int [\max\{P(x,y), \Pi_0(y) + W_0(x) + S(x,y)\} - P(x,y)]v(y')dy' \\
&+ \delta \int [\max\{P(x,y'), \Pi_0(y') + W_0(x)\} - P(x,y)]\gamma(y')dy' \\
&= f(x,y) - \xi S(x,y) - \mu[S(x,y) + W_0(x)] \\
&+ \delta \int [\max\{P(x,y') - \Pi_0(y') - W_0(x), 0\} - P(x,y) + \Pi_0(y') + W_0(x)]\gamma(y')dy' \\
&= f(x,y) - \xi S(x,y) - \mu[S(x,y) + W_0(x)] \\
&+ \delta \int S(x,y')^+\gamma(y')dy' - \delta S(x,y) + \delta \int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy'.
\end{aligned}
$$

Now substitute out $rP(x, y)$ using

$$
\begin{aligned}
rP(x,y) &= rS(x,y) + rW_0(x) + r\Pi_0(y) \\
&= rS(x,y) + b(x) + \kappa \int_{y' \in \mathcal{A}(x)} \big[w_0(x,y') + \beta_0 S(x,y')\big]v(y')dy' - \mu W_0(x) \\
&- c + \delta \int [\Pi_0(y') - \Pi_0(y)]\gamma(y')dy' + \kappa \int_{x' \in \mathcal{B}(y)} \big[(1-\beta_0)S(x',y) - w_0(x',y)\big]u(x')dx' \\
&+ s\kappa \iint_{(x',y') \in \mathcal{C}(y)} \big[S(x',y) - S(x',y') - w_0(x',y)\big]h(x',y')dx'dy',
\end{aligned}
$$

yielding

$$
\begin{aligned}
rS(x,y) + b(x) + \kappa \int\limits_{y' \in \mathcal{A}(x)} \big[w_0(x,y') + \beta_0 S(x,y')\big] v(y') dy' - \mu W_0(x) \\
-c + \delta \int [\Pi_0(y') - \Pi_0(y)]\gamma(y') dy' + \kappa \int\limits_{x' \in \mathcal{B}(y)} \big[(1-\beta_0)S(x',y) - w_0(x',y)\big] u(x') dx' \\
+s\kappa \iint\limits_{(x',y') \in \mathcal{C}(y)} \big[S(x',y) - S(x',y') - w_0(x',y)\big] h(x',y') dx' dy' \\
= f(x,y) - \xi S(x,y) - \mu[S(x,y) + W_0(x)] \\
+\delta \int S(x,y')^+ \gamma(y') dy' - \delta S(x,y) + \delta \int [\Pi_0(y') - \Pi_0(y)]\gamma(y') dy'
\end{aligned}
$$

and rearrange terms to get the surplus equation

$$
\begin{aligned}
(r + \xi + \delta + \mu) S(x,y) = f(x,y) - b(x) - \kappa \int\limits_{y' \in \mathcal{A}(x)} \big[w_0(x,y') + \beta_0 S(x,y')\big] v(y') dy' \\
+c - \kappa \int\limits_{x' \in \mathcal{B}(y)} \big[(1-\beta_0)S(x',y) - w_0(x',y)\big] u(x') dx' \\
-s\kappa \iint\limits_{(x',y') \in \mathcal{C}(y)} \big[S(x',y) - S(x',y') - w_0(x',y)\big] h(x',y') dx' dy' + \delta \int S(x,y')^+ \gamma(y') dy'.
\end{aligned}
$$

This is equation (7) in the text, in which match surplus does not depend on the value of the current wage contract. The match surplus is determined by the fixed point of this equation.

A.3 Solution algorithm

The model is solved by the iterative fixed point algorithm as follows.

1. Fix the number of firms N and solve for the equilibrium surplus $S(x,y)$ and distribution of matches $h(x,y)$ by iterating on the following steps:

   (a) Update the market tightness κ using Eq. (1) for the current distribution of unemployed managers and vacancies.

   (b) Update equilibrium surplus values $S(x,y)$ using match surplus equation (7).

   (c) Use the new surplus values to update the distribution of matches $h(x,y)$ based on the steady flow equation (10).

(d) Calculate the new distributions of unemployed and vacancies $u(x) = l - \int h(x,y)dy$ and $v(y) = N - \int h(x,y)dx$. Use equations (8) and (9) to obtain the number of unemployed U and the number of vacancies V.

(e) Repeat steps (a)-(d) until the surplus function $S(x,y)$, the distribution of matches $h(x,y)$, the distribution of candidates $u(x)$, and the distribution of jobs all satisfy convergence criteria.

2. Check that an equilibrium found above satisfies the free entry condition:

(a) Using equilibrium values of $S(x,y)$ and $h(x,y)$ from step 1, compute the value of outside option for the smallest firm with market value $\Pi_0(y_{min})$ from free entry condition (5).

(b) If the surplus $\Pi_0(y_{min})$ satisfies convergence criteria, move to step 3.

(c) If the surplus $\Pi_0(y_{min})$ does not satisfies convergence criteria, update market tightness κ as

$$\kappa = \frac{c - \delta \int \Pi_0(y')\gamma(y')dy'}{\int\limits_{x' \in \mathcal{B}(y)} \big[(1-\beta_0)S(x',y) - w_0(x',y)\big]u(x')dx' + s \iint\limits_{(x',y') \in \mathcal{C}(y)} \big[S(x',y) - S(x',y') - w_0(x',y)\big]}$$

(d) Using market tightness κ that satisfies free entry condition, find the number of vacancies from Eq. (1) and update the number of firms using $N = V + L - U$ as

$$N = \left(\frac{\eta}{\kappa}\right)^2 \frac{1}{U + s(1-U)} + L - U$$

and the distribution of vacancies as

$$v(y) = n(y) - \int h(x,y)dx.$$

(e) Return to step 1 and find a new equilibrium with updated number of firms N.

3. Using equilibrium values from steps 1 and 2, compute the value of employment contract $W_1(x,y,w)$ by iterating on Eq. (6).